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RESEARCH ON THE DYNAMIC CHARACTERISTICS OF A WATER TOWER

ABSTRACT

Providing the population with high-quality water is becoming an increasingly urgent task for Ukraine at the present time. This is due to both the pollution of existing water sources on the one hand and the constant military threat on the other. Water towers in such conditions perform functions related not only to the transportation of drinking water, but also to its temporary storage. Wind load is the main type of load that determines the dynamic stability of such high-rise structures. However, specialized regulatory and professional literature contains rather limited information on the dynamic behavior of water towers. Therefore, their engineering design does not take into account certain aspects of the structure's operation.

The main purpose of the research is to analyze the dynamic behavior of a water tower using the example of a steel structure located in the city of Mykolaiv. The object considered was an emergency water supply water tower with a steel tank with a diameter of 18 m and a total capacity of 620 m^3. The supporting part of the tower is a central shaft supported by struts. The total height of the tower is 48.7 m. During the research, two main design options proposed by engineers were considered - a variant with additional struts (the main variant) and without additional struts.

The finite element method was used to conduct the research based on the ANSYS 14.5 design and computing complex for Windows with the ANSYS CFD FLUENT additional module.

Analysis of the obtained computer simulation results showed that the design option with the arrangement of the struts is aerodynamically unstable - the limiting frequency of Karman vortex breakdown (0.648 Hz), even with the arrangement of decorative aerodynamic elements, is lower than the calculated one. Therefore, it is proposed to make the tower shaft in the form of a hyperbolic paraboloid of negative Gaussian curvature, which brings the dynamic spectrum of the structure into a safe zone. Such a design solution should be patented.

KEYWORDS: water tower, aerodynamic behavior, vibration resonance, reservoir, shaft, hyperbolic shell

ДОСЛІДЖЕННЯ ДИНАМІЧНИХ ХАРАКТЕРИСТИК ВОДОНАПІРНОЇ БАШТИ

АНОТАЦІЯ

Забезпечення населення якісною водою стає

все більш актуальною задачею для України в теперішній час. Це пов'язано як із забрудненням існуючих джерел води з однієї сторони, так і з постійною військовою загрозою, з іншої сторони. Водонапірні башти в таких умовах виконують функції, пов'язані не тільки із транспортуванням питної води, а й її тимчасовим зберіганням. Вітрове навантаження є основним видом навантажень, яке визначає динамічну стабільність подібних висотних споруд. Разом з тим спеціалізована нормативна та фахова література містить доволі обмежену інформацію стосовно динамічної поведінки водонапірних башт. Тому їх інженерне проєктування не враховує певні аспекти роботи конструкцій.

Основною метою проведених досліджень є аналіз динамічної поведінки водонапірної башти на прикладі сталевої споруди, розташованої в місті Миколаїв. В якості об'єкта розглядалась водонапірна башта аварійного водопостачання із сталевим резервуаром діаметром 18 м і повною ємністю 620 м^3. Опорна частина башти являє собою центральний стовбур, підкріплений підкосами. Повна висота башти становить 48,7 м. В ході досліджень розглядались два основних конструктивних варіанти, запропоновані інженерами – варіант з додатковими підкосами (основний варіант) та без додаткових підкосів.

Для проведення досліджень використовувався метод скінчених елементів на базі проєктно-обчислювального комплексу ANSYS 14.5 для Windows із додатковим модулем ANSYS CFD FLUENT.

Аналіз отриманих результатів комп'ютерної симуляції показав, що конструктивний варіант із розташуванням підкосів виявляється аеродинамічно нестійким – гранична частота зриву вихорів Кармана (0,648 Гц), навіть при влаштуванні декоративних аеродинамічних елементів, нижче за розрахункову. Тому запропоновано стовбур башти виконати у вигляді гіперболічного параболоїда від'ємної гаусової кривизни, що виводить динамічний спектр споруди в безпечну зону. Таке конструктивне рішення має бути запатентовано.

КЛЮЧОВІ СЛОВА: водонапірна башта, аеродинамічна поведінка, вібраційний резонанс, резервуар, стовбур, гіперболічна оболонка

INTRODUCTION

One of the most important factors ensuring the stability of cities in modern conditions of warfare is the creation of a guaranteed water supply system. To this end, it is necessary to organise conditions for providing the population with high-quality water in sufficient quantities for drinking and domestic needs both in the territory of the settlement and for production needs (mechanisation of lifting, transportation and distribution of water, creation and storage of firefighting reserves).

Water is one of the most important elements for humans, and a shortage of it is associated with a direct threat to life. At the same time, there is an important environmental aspect related to the need to provide the population with high-quality water and the possibility of purifying it if necessary. Fifteen years ago, this issue was not particularly acute, as the natural water sources in Ukraine were relatively clean. However, according to available open data [1], the situation has deteriorated sharply over the last decade (Fig. 1). The main role in this process is played by industrial, energy and transport emissions, which lead to the appearance of aggressive chemical, biological and radioactive agents in water sources. All this requires solving the problem of effective water supply to the population, which is quite relevant in today's realities.

To meet the above needs, water towers can be used. A water tower is a structure that regulates all operations of the water supply system, namely, it controls the water supply, pressure and flow. The water tower also creates a reserve of water resources and regulates the operation of the pumping system. Although many such structures have fallen into disrepair [2], their use can be effectively restored.

The main type of load that largely determines the stress-strain state and behaviour of water towers is wind load. According to existing data [3–5], high-rise structures of this type are generally quite sensitive to various dynamic influences. At the same time, the nature of these influences provides opportunities

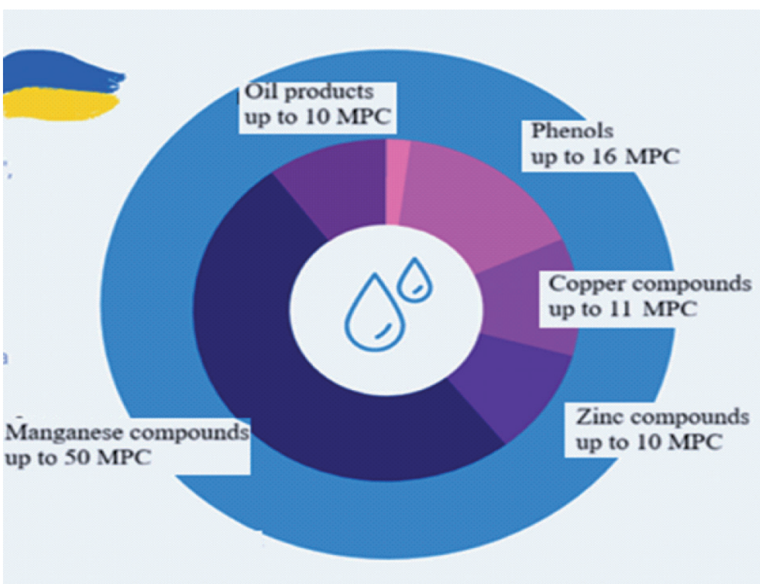


Figure 1 – Pollution of river waters in Ukraine

for optimising the structural form and reducing the material intensity of the structure as a whole.

ANALYSIS OF RECENT STUDIES AND PUBLICATIONS

At present, the issue of wind flow interaction with tall building structures under various conditions is a well-studied area, which is largely considered classic. Among the most well-known works, it is worth noting the works of domestic schools [6–11], as well as the works of foreign specialists [12–14]. However, the main focus is on power line structures. In addition, aspects related to the optimisation of the geometric shape of high-rise building structures are practically not considered.

PROBLEM STATEMENT

In view of the above, the main problem, which is practically not reflected in scientific research, is the analysis of the interaction of wind flow with high-rise building structures such as water towers. This allows not only to assess the technical condition of existing structures, if necessary, but also to predict the effective dynamic performance of the latest design developments.

An important role in this is played by the study of the spectrum of the dynamic characteristics of water towers, which allows the principles of integral dynamic diagnostics of structures to be implemented [15-19].

The main purpose of the work is to conduct numerical studies of the interaction of wind flow with the structure of a water tower.

The object of the study was an emergency water supply tower in the city of Mykolaiv. The tower reservoir consists of an outer ring tank and an inner tank divided into four sectors. For the outer tank: the outer diameter is 18 m, the inner diameter is 9.7 m, and the water column height is 2.3 m. For the inner tank: the outer diameter is 9.7 m, the inner diameter is 2.4 m. The height of the water column is 3.5 m. The volume of water in the outer tank is 410 m^3, in the inner tank – 210 m^3. The total volume of the tank is 620 m^3. The bottom of the tank consists of 10 mm thick sheets reinforced by a system of radial (I-beam No. 40B2) and ring beams (I-beam No. 25B2). The roof is made of 10 mm thick sheets and reinforcing radial beams (I-beam No. 20Sh1).

The supporting part of the tower is a central shaft in the form of a cylinder with variable wall thickness ranging from 12 mm to 20 mm, 4 inclined lower struts (530x12 mm pipe) and 8 upper struts (325x10 pipe). The height of the tower is 44 m. The total height of the tower is 48.7 m. A general view of the tower is shown in Fig. 2.

To check for vortex excitation, the maximum wind speed was calculated, taking into account the data from standard [20]:

$$V_{max} = V_0 C_h = 27.76 \cdot 2.1 = 58.30 \text{ m/s}, \quad (1)$$

where V_0 – the characteristic wind load expressed in terms of velocity (m/s).

Since the frequency characteristics of the tower will be investigated further, it is convenient to express the maximum speed in terms of the maximum Karman vortex shedding frequency:

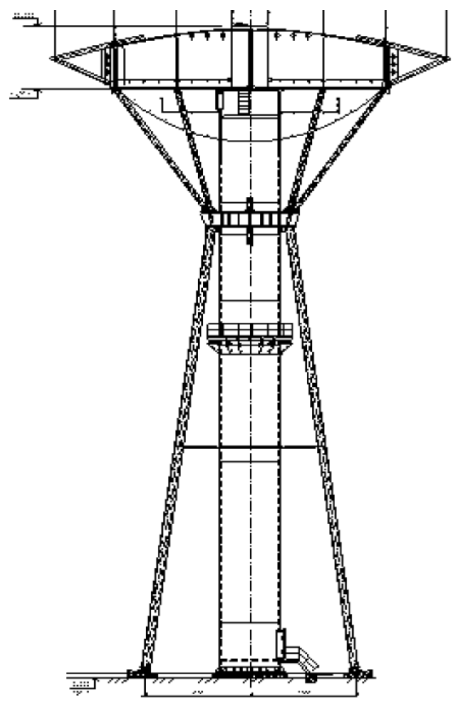


Figure 2 – The water tower under investigation

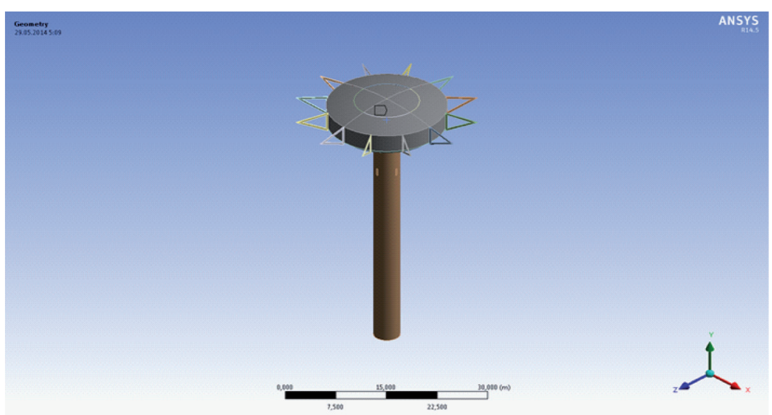


Figure 3 – Calculation model of the tower with a cylindrical shaft

$$f_{k,\max} = \frac{V}{d} Sh = \frac{58.30}{18} \cdot 0.2 = 0.648 \ \text{Hz}, \quad (2)$$

where

d – the largest diameter of the tank;

Sh – Strouhal number.

Thus, the frequency obtained is the limit value at which Karman vortices can form in a given wind area for a cylinder with a diameter of 18 m.

The analysis of the water tower operation was performed using the finite element method, which is very common in modern engineering practice [21, 22]. The ANSYS 14.5 design and calculation complex for Windows was used to conduct numerical studies [23].

MAIN MATERIAL AND RESULTS

During the research, two tower design variants were considered:

- Variant 1: a water tower with a shaft structure in the form of a cylindrical shell (Fig. 3);
- Variant 2: a water tower with a shaft structure in the form of a cylindrical shell with struts according to the design (Fig. 4).

The study took into account the mass of water in the tank, which was modelled using special Flue-type finite elements (Fig. 5). As boundary conditions, rigid fastening was specified at the level of connection of the supporting structures (shaft, struts) to the ground.

To analyse the effect of the static component of wind on the natural frequencies of the structure, turbulent wind flow was modelled. For this purpose, a space with air characteristics was created around the tower (Fig. 5), divided into a total of 1,000,521 finite elements. When choosing a finite element mesh, the author's findings presented in the works [24, 25] were used.

The model was calculated in the ANSYS CFD FLUENT module. The resulting air flow lines are shown in Fig. 7. The pressure on the tower surface is shown in Fig. 8. The results obtained were transferred to the modal analysis as a statically applied load.

As a result of the calculation, the first natural oscillations of the tower fell within the range of up to 0.648 Hz. Therefore, such a structure should be classified as quasi-elastic.

The frequency and first natural

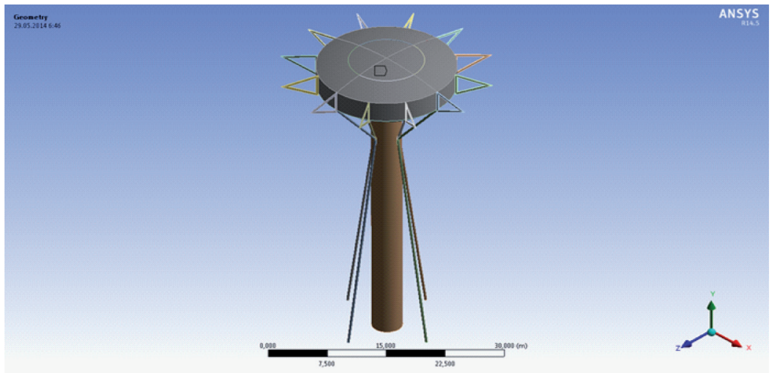


Figure 4 – Calculation model of the tower with a cylindrical shaft and struts

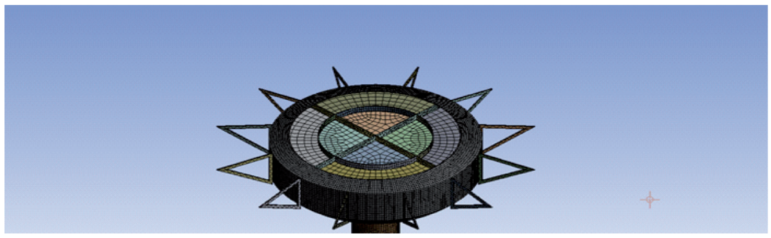


Figure 5 – Finite element model of the mass of water in the tank

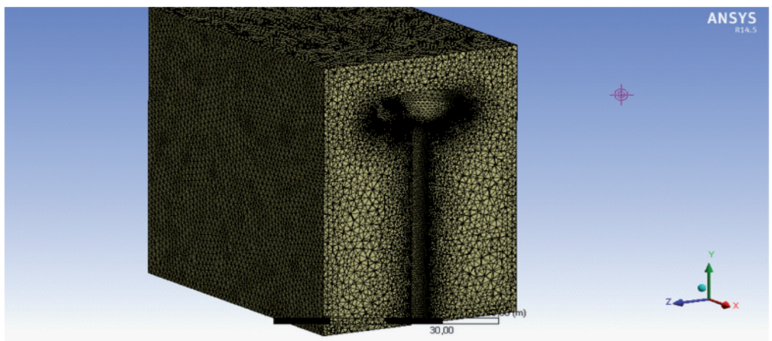


Figure 6 – Finite element model of the tower in wind flow

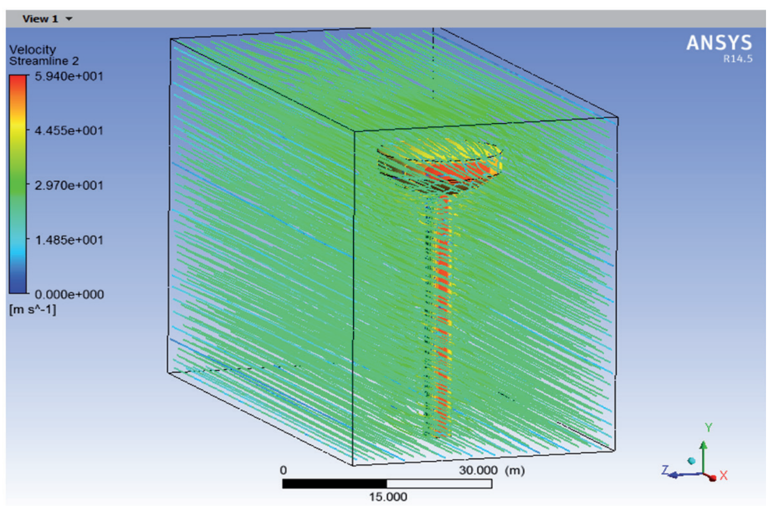


Figure 7 – Current lines, general view

oscillations of the tower with the first variant shaft structure are shown in Fig. 9. The dependence of the oscillation frequency on the change in the studied factors is shown in the graph in Figure 11.

The frequency and first tone of the tower natural oscillations with the second variant shaft are shown in Fig. 10. The dependence of the oscillation frequency on the change in the studied factors is shown in Fig. 12.

INTERPRETATION OF RESULTS AND THEIR APPROBATION

As can be seen from the results obtained, variant No. 2, with a low tank fill level, falls into the dangerous range of Karman vortex shedding, i.e. wind resonance may occur. At the same time, the presence of decorative elements has almost no effect on the change in natural frequencies. This can be explained by their low mass, which does not significantly affect the frequency characteristics of the structure (FCS). The statically applied pressure of the simulated wind flow also does not affect the FCS. It is known that a significant effect on natural frequencies can be exerted by a pre-applied load on thin objects with

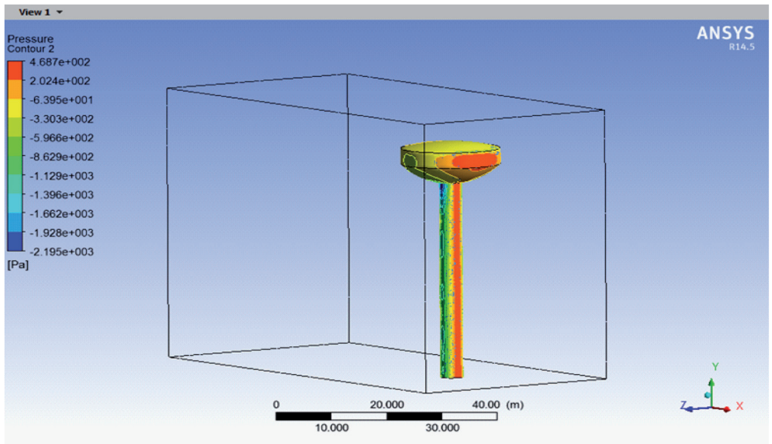


Figure 8 – Wind pressure on the tower

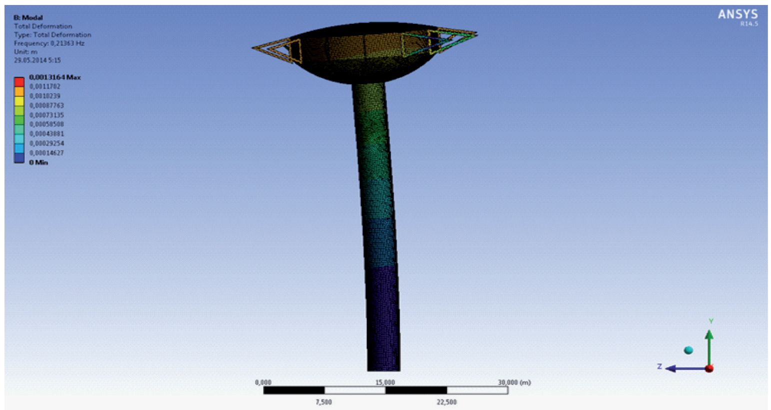


Figure 9 – First form of tower oscillations (variant No. 1)

Table 1– Results of calculations of the frequency spectrum of a water tower

Variant No.	Influencing factors	Mass, cubic metres				
		620	450	300	620	0
		Frequency, Hz				
1	With decorative elements	0.214	0.241	0.281	0.354	0.542
	Without decorative elements	0.217	0.246	0.287	0.369	0.612
	Static wind load	0.217	0.249	0.288	0.375	0.612
2	With decorative elements	0.377	0.425	0.495	0.566	0.764
	Without decorative elements	0.375	0.425	0.503	0.617	0.813
	Static wind load	0.376	0.427	0.503	0.621	0.814
3	With decorative elements	0.683	0.751	0.855	1.038	1.475
	Without decorative elements	0.685	0.754	0.869	1.081	1.513
	Static wind load	0.685	0.756	0.871	1.083	1.517

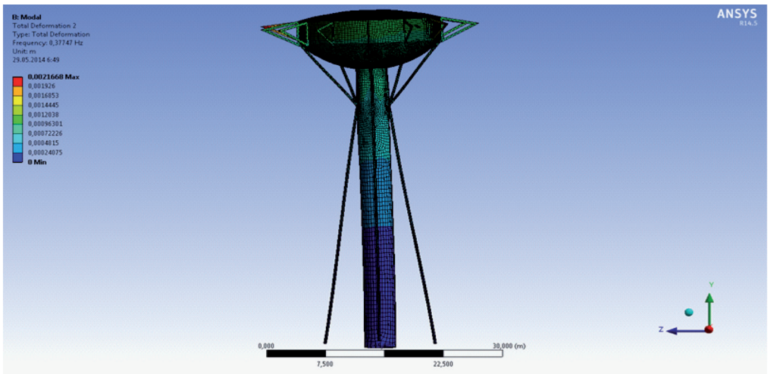
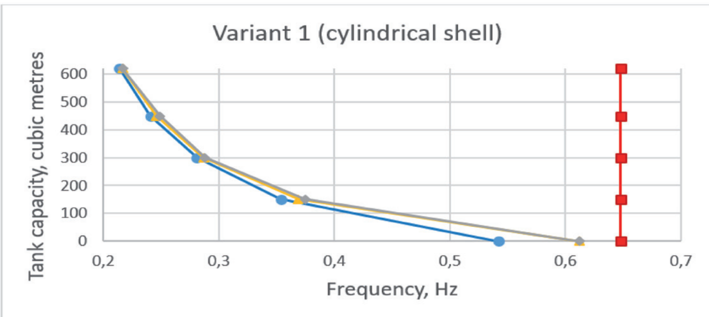
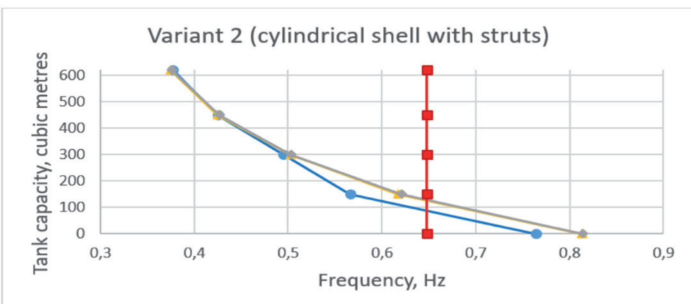


Figure 10 – First form of tower oscillations (variant No. 2)



With decorative elements - Without decorative elements –
Wind impact - Limit frequency of vortex shedding

Figure 11 – Frequency dependence graph (variant No. 1)



With decorative elements - Without decorative elements –
Wind impact - Limit frequency of vortex shedding

Figure 12 – Frequency dependence graph (variant No. 2)

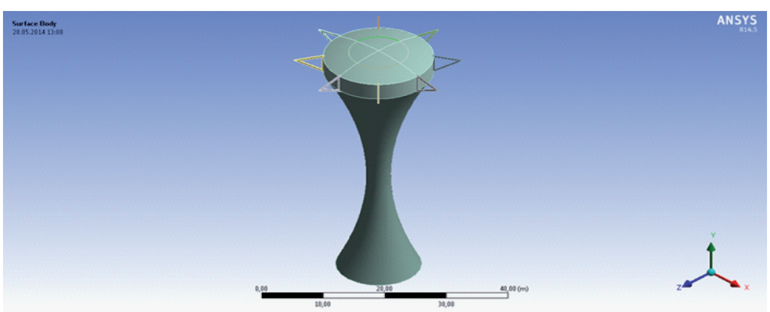


Figure 13 – Calculation model of the tower with a shell-shaped shaft of negative Gaussian curvature (variant No. 3)

very small linear dimensions in one or more directions (e.g., power lines). In our case, the load is applied over a large area across the entire surface of the structure and has relatively small values.

To improve the situation, the possibility of changing the geometry of the water tower shaft to a hyperboloid of negative Gaussian curvature was analysed (Fig. 13). As it turned out, this design (variant No. 3) showed significantly better results than the previous two variants (Fig. 14). A summary table of the calculation results is presented in Table 1. Thus, the water tower under consideration was removed from the dangerous range of vortex shedding.

CONCLUSIONS AND PROSPECTS FOR FURTHER DEVELOPMENTS

Based on the research conducted, the following conclusions can be drawn:

1. The current method of designing water towers does not pay sufficient attention to possible aerodynamic effects. Therefore, the task of analysing aerodynamic stability is quite relevant.
2. Based on computer modelling, the dynamic characteristics for a 50 m high water tower were determined and it was established that the existing structure is not fully aerodynamically stable.
3. The design approach to changing the shape of the tower shaft proposed in the work made it possible to significantly improve the aerodynamic qualities of the object under consideration, including at different levels of water filling of the structure.
4. This design solution is planned to be patented, and the approach developed in the work will be taken into account in the methodology for designing water towers.

A patent application is being filed for the developed design solution. The formula of the invention: the supporting part of the water tower is a shaft, which has the form of a closed shell and is distinguished by the fact

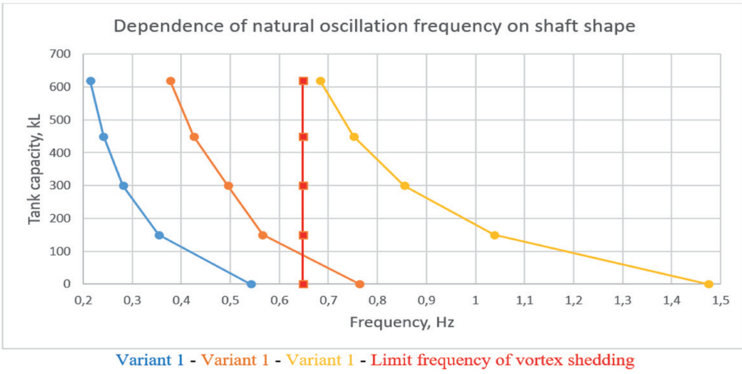


Figure 14 – Comparison of design variants

that in order to increase the frequency of the natural oscillations of the structure, the shaft is made in the form of a solid hyperboloid of negative Gaussian curvature.

The approach developed in the course of the presented research can be taken into account in the methodology for designing water towers. Separate provisions should be included in the current standards for designing high-rise building structures.

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