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AN ASSESSMENT METHOD FOR THE CONTROL SYSTEMS QUALITY

МЕТОД ОЦІНКИ ЯКОСТІ СИСТЕМ КЕРУВАННЯ

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Abstract*. Possibilities and methods of applying the concept of uncertainty in order to assess the quality of control are investigated. An analysis of the approaches currently used for uncertainty assessment is carried out. The use of the informational approach for this purpose is substantiated. It is proposed to use informational uncertainty as a criterion for the quality of control tools. For this, the amount of negative information (misinformation) caused by the imperfection of management methods and devices is calculated. The method of estimating the amount of misinformation is based on Bongard's concept of uncertainty. Misinformation is considered as Bongard's negative useful information. The amount of misinformation is the difference between the Shannon entropy and the Bongard's uncertainty and is used as a criterion for absolute information uncertainty. The criterion of relative information uncertainty is also proposed as the ratio of the amount of misinformation introduced by the control tool to the maximum possible value of misinformation. The maximum value is the amount of misinformation at zero Shannon entropy. Mathematical expressions for calculating the absolute and relative uncertainty of control systems are given.*

Formulas for calculating deterministic analogs of Shannon's entropy and Bongard's uncertainty are proposed to assess the quality of control tools that are investigated by non-statistical methods. Appropriate expressions for calculating criteria of absolute and relative uncertainty based on transient processes of control systems are derived.

The practical use of the proposed method is shown. To demonstrate the use of the criterion of information uncertainty, simulation of the PID controller was carried out using Scilab/Xcos tools. The vectors of input and output values obtained as a result of modeling were processed using the formulas introduced in this article. The criterion of relative information uncertainty was applied to compare the quality of PID controllers that were discretized by different methods.

Анотація*. Досліджуються можливості та методи використання концепції невизначеності з метою оцінки якості управління. Виконаний аналіз підходів, що нині використовуються для оцінки невизначеності. Обґрунтовується використання з цієї метою інформаційного підходу. Пропонується використовувати інформаційну невизначеність як критерій якості засобів керування. Для цього розраховується кількість негативної інформації (дезінформації), що спричиняється недосконалістю методів та пристроїв керування. Метод оцінки кількості дезінформації заснований на концепції невизначеності Бонгарда. Дезінформація розглядається як негативна корисна інформація Бонгарда. Кількість дезінформації є різницею між ентропією Шеннона і невизначеністю Бонгарда і використовується як критерій абсолютної інформаційної невизначеності. Запропонований також критерій відносної інформаційної невизначеності як відношення кількості внесеної засобом керування дезінформації до її максимально можливого значення. За максимальне значення береться кількість дезінформації при нульовому значенні ентропії Шеннона. Наведені математичні вирази для розрахунку абсолютної і відносної невизначеності систем керування.*

Для оцінки якості керуючих засобів, які досліджуються нестатистичними методами, запропоновані формули для розрахунку детермінованих аналогів ентропії Шеннона і невизначеності Бонгарда. Виведені відповідні вирази для розрахунку критеріїв абсолютної і відносної невизначеності на основі перехідних процесів систем керування.
Показане практичне використання запропонованого методу. Для демонстрації використання критерію інформаційної невизначеності було проведено моделювання ПІД-регулятора засобами Scilab/Xcos. Отримані в результаті моделювання вектори значень вхідних і вихідних величин оброблялись за допомогою введених у цій статті формул. Критерій відносної інформаційної невизначеності був застосований для порівняння якості ПІД-регуляторів, які були дискретизовані різними методами.
Key words: control system, criterion, quality of control, uncertainty measure, Entropy, Bongard's uncertainty, misinformation, information saturation, Scilab/Xcos.
Ключові слова: система керування, критерій, якість контролю, міра невизначеності, ентропія, невизначеність Бонгарда, дезінформація, інформаційна насиченість, Scilab/Xcos.

Introduction. During the analysis and synthesis of control systems, the task of assessing the effectiveness and quality of systems arises. Today, there are many criteria of effectiveness and quality, which are often contradictory. To assess the quality of control, the following are used: overshoot, oscillation, duration of the transient process, settling time, time to reach the first maximum of the controlled value, stability margin, response speed, frequency of natural oscillations of the system, etc. Improving one indicator leads to deterioration of another. It would be more convenient to have a single universal criterion that provides a comprehensive assessment of the quality of control tools.
Thus, there is a need to develop a universal criterion for assessing the accuracy of the production of control influences, which would allow taking into account various factors affecting the quality of the designed control system.
It is known that a high degree of generalization of patterns and phenomena in a wide variety of areas is achieved by using methods and concepts of information theory. This allows one to abstract from specific physical processes occurring in the designed system. Many researchers view information as a methodological basis for generalization and simplification.
It is necessary to analyze information processes, estimate the amount of information circulating in the system, and calculate the degree of its distortion. Such characteristics of control systems as complexity, orderliness, organization and entropy are used. However, such approaches have not found wide application in the practice of analysis and synthesis of control systems.
Lately the Uncertainty Approach (UA) has become established in measurement theory. But uncertainty is not only found in measurements. To a large extent, this concept also applies to problems of control. The expediency of using UA also arises in relation to the quality of controls.
In paper [1], a similar problem was solved for assessing the quality of measuring instruments. It was proposed to use the information criterion. There is introduced the concept of information uncertainty, which is estimated by the amount of negative useful information, that is, misinformation introduced by the measuring instrument. This approach is also useful for solving problems of analysis and synthesis of control systems.
Literature review. Since Shannon introduced the concept of information entropy as a quantitative measure of the uncertainty of some source of information, a large number of researchers have proposed a number of other approaches to estimate the uncertainty.
The first push for this was made by Alfrped Renyi in his report at the 4th Berkeley Symposium [2]. He considered the problem of estimation of amount of uncertainty of the distribution $\wp$, that is, the amount of uncertainty concerning the outcome of an experiment, the possible results of which have the probabilities p_1, p_2, …, p_n. Renyi pointed out that the Shannon entropy

$$H_\alpha(\wp) = \sum_{k=1}^{n} p_k log_2 \frac{1}{p_k} \tag{1}$$

is characterized by the following postulates:
(a) $H(p_1, p_2, \ldots, p_n)$ is a symmetric function of its variables for n = 2, 3,…
(b) $H(p, 1–p)$ is a continuous function of p for $0 \le p \le 1$.
(c) $H(1/2, 1/2) = 1$.
(d) $H[\wp * \mathcal{Q}] = H(\wp) + H(\mathcal{Q})$ for two probability distribution $\wp = (p_1, p_2, \ldots, p_n)$ and $\mathcal{Q} = (q_1, q_2, \ldots, q_n)$.

Renyi indicated that there are many quantities other than (1) which satisfy the postulates above. He suggested using the next quantity as a measure of the entropy of the distribution $\wp = (p_1, p_2, \ldots, p_n)$:

$$H_\alpha(\wp) = \frac{1}{1-\alpha} log_2 \left(\sum_{k=1}^{n} p_k^\alpha \right),$$

where $\alpha > 0$ and $\alpha \neq 1$. He called it the entropy of order α of the distribution $\wp$.

The authors of the article [3] made a review of entropy measures for uncertainty quantification that appeared after Renyi, such as Tsallis entropy, Sample entropy, Permutation entropy, Approximate entropy, and Transfer entropy. It is shown here that the information is the decline in disorder and ambiguity, uncertainty is referred to the unlikelihood of logical reasoning, entropy is the expected information, and ignorance is the lack of knowledge regarding the uncertainty.

Classical information theory is based on the use of probabilistic characteristics. As it shown in [4], evidence theory is able to better handle unknown and imprecise information. Owing to its advantages, evidence theory has more flexibility and effectiveness for modeling and processing uncertain information.

The essence of the evidence theory is described in sufficient detail in [5]. Evidence theory extends classical probability theory. It is based on the basic probability assignment concept (b.p.a.), a generalization of the concept of the probability distribution in probability theory. Each b.p.a. in evidence theory has a belief function and a plausibility function associated with it. The belief (plausibility) value of a set is the minimum (maximum) support of information represented by the b.p.a. on that set.

As evidence theory generalizes probability theory, there are more types of uncertainty in evidence theory than in probability theory. In particular, Yong Deng [6] proposed a new uncertainty measure named as Deng entropy:

$$E_d(m) = -\sum\nolimits_{A \in X} m(A) log_2 \frac{m(A)}{2^{|A|} - 1},$$

where m is a mass function. If Y is a set of mutually exclusive and collectively exhaustive events denoted as $Y = \{\theta_1, \theta_2, \ldots, \theta_i, \ldots, \theta_{|Y|}\}$, then mass function is a mapping m from 2^Y to [0, 1], defined as

$$m{:}2Y \rightarrow [0,1]$$

and satisfying the following conditions:

$$m(\emptyset) = 0, \sum_{A \in 2^Y} m(A) = 1.$$

Deng entropy is the generalization of Shannon entropy since the value of Deng entropy is identical to that of Shannon entropy when the b.p.a. defines a probability measure.

Paper [7] deals with the assessment of randomness of a certain variable X and with different criteria that can be used to evaluate randomness. There are established optimal bounds between an tropic quantities and statistical quantities in an interplay between information theory and statistics.

Article [8] states that there are situations where probabilistic measures of entropy do not work. To deal with such situations, instead of taking the probability, the idea of fuzziness can be explored. In this paper, a two-parameter generalized measure of fuzzy entropy is proposed, where instead of the probability $p(x_i)$ the membership function $\mu(x_i)$ of the fuzzy set is used:

$$H_\alpha^\beta(A) = \frac{\beta}{1-\alpha} \log_D \left[\frac{1}{n} \sum_{i=1}^{n} \left\{ \mu_A^{\beta(1-\alpha)}(x_i) + (1 - \mu_A(x_i))^{\beta(1-\alpha)} \right\} \right]; \ 0 < \alpha < 1, 0 < \beta \le 1; \beta > \alpha$$

This value satisfies all the properties of fuzzy entropy, thus it is a valid measure of fuzzy entropy.

In study [9] it is proposed a new intuitionistic fuzzy entropy-based algorithm for feature selection before classification tasks in an information system. For this purpose, new intuitionistic fuzzy entropy has been developed to measure feature entropy (uncertainty) as parameters for feature selection.

A certain contribution to the development of uncertainty assessment methods was made by Ukrainian researchers. In report [10] it is proposed an information model of the functioning of advertising. There is introduced ideas about useful and harmful (excess) information. The concept of user’s thesaurus is also introduced. The effectiveness of advertising is determined by the mutual influence of useful and redundant information. The effectiveness of advertising is determined by the perception of positive information by the recipient. Analysis of the information model of advertising leads to the conclusion of a two-stage process of functioning of advertising. This is due to the influence of redundant information on useful information that should be perceived by the recipient. In the case of the advertising operation the redundant information is always harmful.

Let $I_1(t)$ and $I_2(t)$ be the functions of accumulation of positive and negative information:

$$\frac{dI_1}{dt} = T - \beta I_2;$$
$$\frac{dI_2}{dt} = -\beta' I_1 - T,$$

where T is a thesaurus of recipients; β is a coefficient of influence of the negative information on the positive one and β' is a coefficient of influence of the positive information on the negative one. It is presumed that β, $\beta' < 1$.

Solving these equations, we obtain:

$$I_1 = \frac{T}{\beta\beta'}\{e^{\beta\beta' t} - 1\};$$
$$I_2 = \frac{T}{\beta}\{1 - e^{\beta\beta' t}\},$$

For large enough t:

$$I_1 = \frac{T}{\beta\beta'}\{e^{\beta\beta' t}\};$$
$$I_2 = -\frac{T}{\beta}\{e^{\beta\beta' t}\}.$$

These results show that an accumulation of both kinds of information $I_1(t)$ and $I_2(t)$ depends on their mutual influence.

Based on the given expressions, the criterion of advertising effectiveness is obtained:

$$\delta = \frac{e^{\beta\beta' t}}{e^{\beta\beta' t} + \frac{T}{\beta}}.$$

The article [11] deals with the analysis of reliability and objectivity of information that can be found on the internet and the objectivity and reliability of such information is compared to the system's behavior. The terms "useful" and "useless" information have been introduced. On the basis of Shannon's law of connection between information and entropy, as the measure of system's organization the notion of information chaos is analyzed, it illustrating growth of entropy in such system.

Described is the parameter which has to discern authentic useful information available for analyzing and obtaining new knowledge from false and biased. A variant of the general scheme of the dynamic information system, reflecting the appearance of inaccurate information, is given.

The vast majority of the considered articles represent exclusively theoretical developments. As an example of the practical application of information approaches, we can cite [12, 13].

The article [12] provides examples of the practical use of the entropy approach to statistical hypothesis testing problems. The author considers relative entropy as the difference between two hypotheses H_0 and H_1. The Kullback-Leibler discrepancy is used as a measure of the discrepancy between the two distributions $\wp = (p_1, p_2, \ldots, p_n)$ and $\mathcal{O} = (q_1, q_2, \ldots, q_n)$:

$$D_{KL}(\wp || \mathcal{O}) = \sum p log\left(\frac{p}{q}\right).$$

It is also argued that the relative entropy is a special case of the Renyi entropy of order $\alpha \rightarrow 1$.

Another option is the Jeffries divergence as a symmetric version of the Kullback-Leibler divergence;

$$D_J(\wp || \mathcal{O}) = D_{KL}(\wp || \mathcal{O}) + D_{KL}(\mathcal{O} || \wp).$$

For distributions $\wp = (p_1, p_2, \ldots, p_n)$ and $\mathcal{O} = (q_1, q_2, \ldots, q_n)$:

$$D_J(\wp || \mathcal{O}) = \sum (p - q) log\left(\frac{p}{q}\right).$$

As to [13] the structural reliability assessment is considered here for the larger and more complex systems with implicit performance functions, also called black-box problems.

Kriging is a widely used and accurate interpolation method, that involves constructing a meta-model to approximate the existing computer simulation model through the design of experiments and a small amount of simulation model calculation. Using the active learning strategy, the Kriging model is iteratively updated until the desired level of accuracy is achieved and deemed satisfactory.

This article aims to the improvement of efficiency for training Kriging metamodel by the proposed learning function based on information entropy theory, namely IE-AK. By progressively exploring these important samples to achieve the trade-off between accuracy and efficiency, the new method greatly accelerates the convergence of the Kriging metamodel without loss of accuracy, which is validated by a series of numerical examples.

Purpose and Objectives. The purpose of this report is to justify the application of the information uncertainty criterion as a generalized criterion for the quality of control systems. This necessitates solving the following problems:
- to formulate a precise statement of the task;
- to develop a quality assessment method based on the use of information criteria;
- to demonstrate the practical use of the described criteria.

Methods. The methods of the theory of automatic control, mathematical modeling, and mathematical statistics were used during the research. Open source software for numerical computation Scilab was used as a tool. Modeling of systems was carried out by the imitative simulation system Xcos.

Results and Discussion. The design and operation of control systems are carried out under conditions of uncertainty caused by incomplete knowledge of the processes occurring in the system, unreliability of technical and software tools, etc. The generation of control actions is carried out by the control device based on information about the state of the control object and/or about external disturbances acting at this object.

Let us consider a typical structural diagram of the control process (Fig. 1).

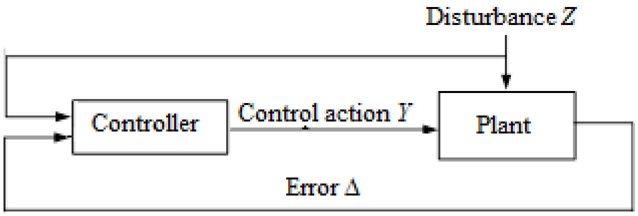

Figure 1 – Structural diagram of the control process

The control device generates the control action Y based on the input information, which is either data on the deviation Δ of the state parameter of the control object from the specified value, or data on the disturbance Z acting at the object. For generalization, we denote the input of the control device as X. Thus, the control device performs the functional transformation $Y = f(X)$.

Let us assume that $P = \{p_i\}$ is the probability distribution of input X, and $Q = \{q_i\}$ is the probability distribution of output Y. For an ideal control device, $P = Q$. In reality, these distributions do not coincide due to the imperfection of the device. The degree of mismatch can be estimated by the Bongard uncertainty value [14]:

$$N(p/q) = -\sum_{i=1}^{n} p_i \log q_i.$$

According to Gibbs' theorem, the inequality

$$-\sum_{i=1}^{n} p_i \log p_i \leq -\sum_{i=1}^{n} p_i \log q_i$$

is satisfied for all probability distributions $\langle p_i \mid i\in\mathbb{N}_n \rangle$ i $\langle q_i \mid i\in\mathbb{N}_n$ and for all $n\in\mathbb{N}_n$; the equality holds if and only if $p_i = q_i$ for all $i\in\mathbb{N}_n$. The proof of the theorem is given in [15].

In general,

$$N(p/q) \geq N(p/p) = -\sum_{i=1}^{n} p_i \log p_i = H(p),$$

where $H(p)$ is Shannon's information entropy.

It is known that the amount of information is a measure of uncertainty reducing regarding the object under study. Bongard introduced the concept of useful information contained in the hypothesis q, with respect to a problem with an answer probability distribution p. If it is assumed that useful information is zero when $q_i = 1/n$, the increment in useful information is given by the expression:

$I = \log n - N(p/q)$.

Since, due to the imperfection of the control device, the Bongard uncertainty increases relative to the entropy H(p), it can be argued that such a device introduces misinformation in the amount

$$D = N(p/q) - H(p) = -\sum_{i=1}^{n} p_i \log \frac{p_i}{q_i}.$$

Following the findings of the article [1], we will use the last expression as a measure of the information uncertainty (IU) of control devices. We will also use the relative information uncertainty (RIU) as the ratio of IU to its maximum value, which occurs at H (p) = 0:

$$\nu = \frac{D}{N(p/q)} = 1 - \frac{\sum_i p_i \log p_i}{\sum_i p_i \log q_i} = 1 - \frac{H(p)}{N(p/q)}. \qquad (2)$$

For control systems in which inputs and outputs are continuous quantities, sums are replaced by integrals, and discrete probabilities are replaced by probability densities. Shannon entropy is replaced with differential entropy:

$$h(x) = \int_{-\infty}^{\infty} f(x) \log f(x)\, dx.$$

Instead of Bongard's uncertainty we will use the following expression:

$$h(x/y) = \int_{-\infty}^{\infty} f(x) \log f(y)\, dx.$$

So, the formula for calculating RIU will take the following form:

$$\nu = 1 - \frac{h(x)}{h(x/y)}.$$

The parameters of discrete and continuous distribution laws can be obtained through statistical experiments.
Relative information uncertainty has the following properties:

- for an ideal control device RIU is zero;
- since $H(p) \le N(p/q)$, RIU is a non-negative value;
- RIU approaches 1 when $N(p/q) >> H(p)$.

Using RIU makes it possible to compare the quality of various tools and methods of control to select the optimal option.
In addition, RIU has the following advantages:

- it provides an assessment of the quality of control using one generalized criterion;
- it considers the laws of distribution of input and output signals, which also makes it possible to evaluate how a particular device is suitable for a specific input value X;
- it does not require complex mathematical calculations.

The quality of the controllers' operation is usually assessed by observing transient processes, i.e. the change in the output value $Y(t)$ that occurs when test signals $X(t)$ are supplied to the controller. In this case, there is no possibility of directly applying the relationships described above. It becomes necessary to assess the amount of information (and misinformation) using non-statistical methods. To do this, we will introduce a number of analogs for the quantities used in calculating information criteria. We will assess the information saturation of the signal $Y(t)$ using the intensity of its changes:

$$\pi_y(t) = \frac{\left| {dy}/{dt} \right|}{\varepsilon_y},$$

where ε_y – resolution threshold of $Y(t)$ values, such as the quantization step.
As an analogue of the Shannon entropy, we will use the integral characteristic of the variability of the value $Y(t)$:

$$\upsilon_H = -\frac{\int_T \pi_y(\theta)\, log\left[\pi_y(\theta)\varepsilon_t\right] d\theta}{\int_T \pi_y(\theta) d\theta},$$

where $\theta = t/T$ – relative time; T – time interval of observation of the variable $Y(t)$; ε_t – time resolution threshold, for example, the sampling interval of the signal in time.
An ideal regulator would be able to instantaneously track changes in the test signal $X(t)$. We denote the output of such an ideal regulator by $\breve{Y}(t)$. Then, as an analogue of Bongard's uncertainty, we will use the following quantity:

$$\upsilon_B = -\frac{\int_T \pi_y(\theta)\, log\left[\pi_{\breve{y}}(\theta)\varepsilon_t\right] d\theta}{\int_T \pi_y(\theta) d\theta}.$$

For practical calculations, it is convenient to use discrete forms:

$$\upsilon_H = -\frac{\sum_i \pi_{yi}\, log\, \pi_{yi}}{\sum_i \pi_{yi}},$$

$$\upsilon_B = -\frac{\sum_i \pi_{yi} \log \pi_{\breve{y}i}}{\sum_i \pi_{yi}},$$

where π_{yi} and $\pi_{\breve{y}i}$ are values of the intensity of change in the outputs of the real and ideal regulators, determined for discrete moments of time θi. RIU criterion:

$$\nu = 1 - \frac{\upsilon_H}{\upsilon_B}. \quad (2)$$

To demonstrate the use of the criterion of information uncertainty, we will perform PID controller simulation using Scilab/Xcos. The model is presented in fig. 2.

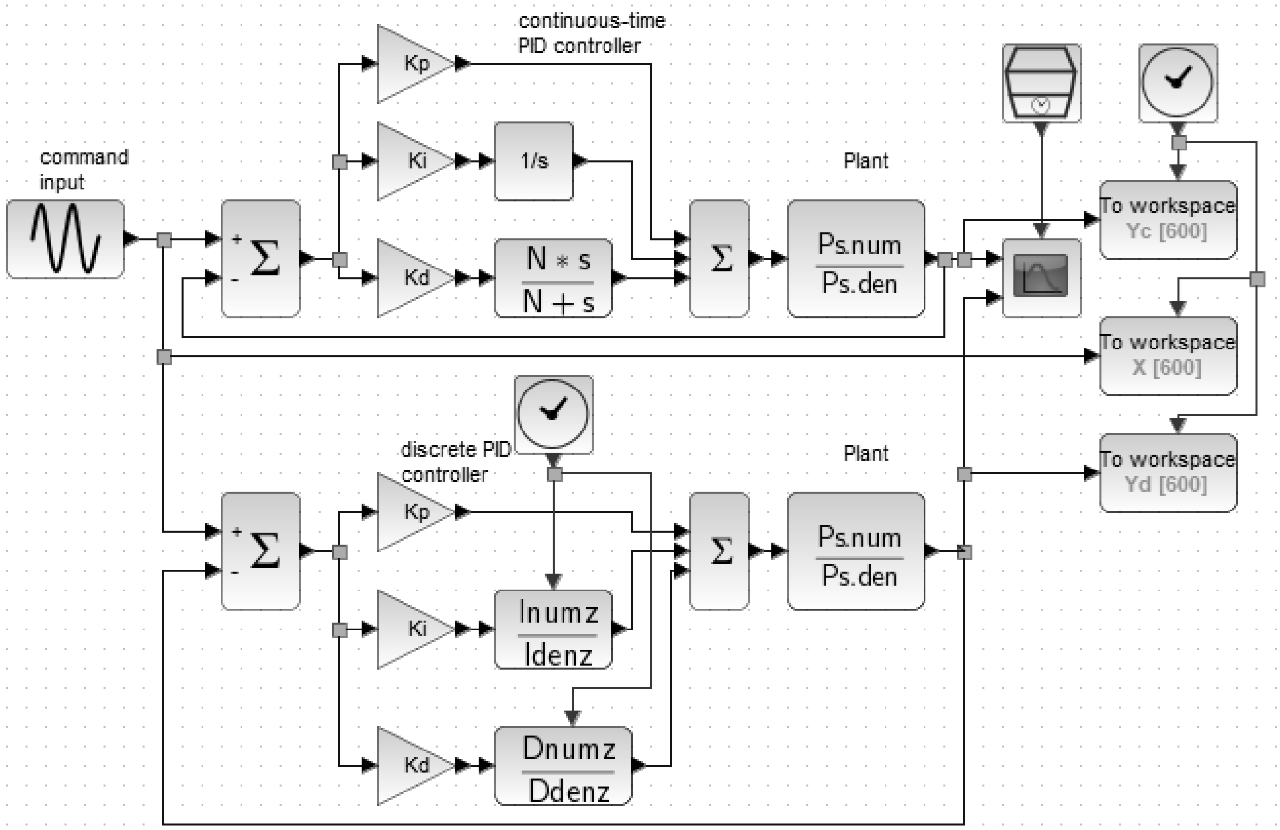

Figure 2 – Xcos-model

In the upper part of the diagram, a model of a continuous-time PID controller is presented, in the lower part, a discrete one. Vectors of input and output values are transferred to the Scilab workspace. These data are processed according to formula (2) by the following Scilab-scenario:

```
pid=abs(diff(Yd.values));
piX=abs(diff(X.values));
Sh = -sum(pid.*log2(pid))/sum(pid);
Bo = -sum(pid.*log2(piX))/sum(pid);
critd = 1-Sh/Bo;
disp(critd)
```

Discretization of the PID controller was performed by two methods: by Euler and by Tustin. For each of these options, the simulation was run and the amount of misinformation was calculated. The results in Table 1 show the better quality of the controller discretized by Tustin's method. Which corresponds to the theory.

Table 1 – Research results

Controller	The amount of misinformation
discretized by the Euler's method	0.0467428
discretized by Tustin's method	0.0565175

Conclusion. The suggested method of the control quality assessment, which is based on the application of information criterion, allows for a simple and effective evaluation of the accuracy and quality of technical control devices.

The generalized criterion of control quality can be the information uncertainty of the control action, based on the use of Bongard's uncertainty. The uncertainty assessment criterion is defined as the ratio of the amount of misinformation introduced by the control device to the maximum possible amount.

The proposed information criterion can be effectively used both for assessing the quality of existing control systems and for selecting methods for increasing the accuracy of designed systems.

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