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# Study of the efficiency of a gas turbine on CO<sub>2</sub> for the use of secondary heat of mining enterprises

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**Annotation.** The mining industry has significant sources of secondary thermal resources with low temperatures of 35°C- 80°C. For their use in the production of additional electricity, it is proposed to use a gas turbine that operates on carbon dioxide (freon R-744) and implements the Rankine cycle. Known detailed tables and model equations for carbon dioxide describe its thermophysical properties. However, they are very complex and challenging to access for inspection. Therefore, this article aims to develop simple analytical expressions for the thermophysical parameters of R-744 for temperatures of 3°C- 80°C and use them to study effective operation modes of a gas turbine for the use of secondary heat resources of mining enterprises. Mathematical modelling methods, theories of approximation of functions, fundamental laws of gas dynamics and technical thermodynamics were used to achieve the goal. The study resolved the issue of selecting an effective turbine operating mode based on the level of thermodynamic efficiency based on general thermodynamic laws. It was shown that reducing the cooling temperature of R-744 in the condenser from 20°C to 5°C due to the use of mine water increases the thermodynamic efficiency of the Rankine cycle almost twice.

## 1. Introduction

The most complete and effective use of secondary energy resources in various branches of industry is one of the leading directions of prospective development. The mining industry has significant sources of secondary thermal resources with low and low-temperature potential. We are talking about the temperature range from 35 to 80°C. This thermal resource can be attracted from underground horizons based on experience in the thermal regimes of deep wells [1]. For their use in generating additional electricity, a gas turbine that works on carbon dioxide and implements the Rankine cycle is proposed. Carbon dioxide belongs to natural freons with a very low global warming index, does not affect the ozone layer and is a non-flammable gas. These favourable properties are significant for its use in the mining industry.

There are detailed tables [2] and model equations for describing the thermophysical properties of CO<sub>2</sub> for carbon dioxide in different phases. However, the model equations are complex and challenging to access for inspection. However, they refer to an extensive range of thermodynamic parameters. In terms of temperature, the operating range of a CO<sub>2</sub> gas turbine is already considerable.



We specified its parameters above. However, at the lower limit, it can be expanded to 3-5 °C due to mine water's use to cool the turbine condenser.

Therefore, this article aims to develop simple analytical expressions for the thermophysical parameters of CO<sub>2</sub> for temperatures of 3-80°C and to use them to study effective operation modes of a gas turbine for the use of secondary heat resources of mining enterprises.

## 2. Method

The mathematical modelling methods, approximation theory and the fundamental laws of gas dynamics and technical thermodynamics were applied to the study. The data analysis and generalisation methods of known theoretical propositions and experimental facts were also used.

## 3. Results and discussion

When constructing simple analytical expressions for the thermophysical parameters of CO<sub>2</sub> in the temperature range of 3°C- 80° C, we use reference data on the thermophysical properties of R-744 [2]. To analyse the change in thermophysical parameters and to select acceptable mathematical expressions, graphical visualization of reference tabular data was carried out. After that, quite simple and close in their analytical structure expressions were proposed for modelling the main thermophysical properties - enthalpy, entropy, heat of vaporization for R-744.

Since the main task is related to the use of low-potential heat for the production of additional electrical energy during the operation of the gas turbine, the pressure range was from  $p_2 = 7.2137$  MPa to  $p_1 = 3.9695$  MPa. This corresponds to the temperature range of R-744 saturated steam from 5°C to 30°C. But for the continuity of changes in the parameters of the working medium R-744 in the Rankine cycle for a steam turbine, the working medium must be superheated to a temperature of the order of 50°C – 60°C. This explains the range of temperatures in our study.

For the R-744 vapor enthalpy depending on the temperature at the isobar of 7.2137 MPa, the expression is proposed

$$H(T) = 21945 \cdot (T - 303)^{0.45} + 365129, \quad (1)$$

where  $H$  is steam enthalpy, J/kg,

The entropy of a supersaturated R-744 vapor under the same conditions as (1) is modelled by the following expression

$$S(T) = 77 \cdot (T - 303)^{0.43} + 1543, \quad (2)$$

where  $S$  – is the enthalpy of steam, J/(kg·K),

The heat of vaporization of R-744 depending on the temperature in the temperature range from 5°C to 31.18°C is modelled by the expression

$$L_{744}(T) \cdot 10^{-3} = (T_{c744} - T)^{0.3} + 440 \cdot (T_{c744} - T)^{0.525} - (T_{c744} - T), \quad (3)$$

where  $L_{744}$  is the heat of vaporization of R-744, J/kg,  $T_{c744}$  is the critical temperature of freon R-744, which is equal to 304.33 K (31.18°C).

For the specific volume of supersaturated R-744 vapor depending on the temperature at the isobar of 7.2137 MPa, the expression is proposed

$$\nu(T) = \left[ 0.6 \cdot (T - 303)^{0.5} + 2.898 \right] \cdot (10^{-3}), \quad (4)$$

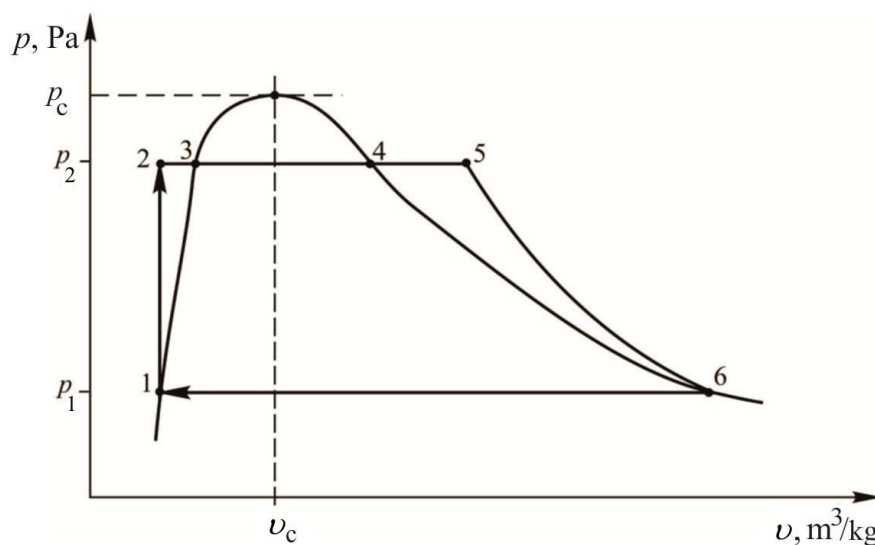
where  $\nu$  – is specific volume, m<sup>3</sup>/kg.

We use the obtained expressions when simulating the movement of R-744 steam in the nozzle apparatus of the turbine. The speed of steam leaving the nozzle apparatus is determined by Bernoulli's equation

$$H(T_5) + \frac{C_0^2}{v_5 \cdot 2} = H(T_1) + \frac{C_6^2}{v_{6,s}(T_1) \cdot 2 \cdot \varphi_a} \quad (5)$$

$T_5$  – the temperature of the working body at point "5" in figure 1, K;  $C_0$  – the speed of the gas flow at the entrance to the nozzle apparatus, (m/s);  $v_5$  – of the specific volume of supersaturated R-744 steam at the inlet to the nozzle apparatus, ( $\text{m}^3/\text{kg}$ );  $\varphi_a$  – coefficient of gas flow speed at the exit from the nozzle apparatus, which is determined by the total coefficient of energy loss from friction, at the exit edges and at the ends of the blades along their height (length) [3];  $T_1$  – the temperature of the working body on the line "1-6" in figure 1, K;  $C_6$  – the speed of the gas flow at the outlet of the nozzle device, (m/s);  $v_{6,s}(T_1)$  – specific volume of saturated R-744 vapor at the outlet of the nozzle device, ( $\text{m}^3/\text{kg}$ ).

Figure 1 shows the points of the Rankine cycle for which the values in equation (5) and other subsequent equations were used.



**Figure 1.** Rankine cycle in "pressure  $p$  - specific volume  $v$ " coordinates, where  $p_1$ ,  $p_2$  - freon pressure on lines "6-1" and "2-5", respectively, Pa;  $p_c$ ,  $v_c$  - pressure (Pa) and specific volume ( $\text{m}^3/\text{kg}$ ) at the critical point [4].

The index  $n$  of the polytropic process of gas movement in the nozzle is determined by the equation of the first beginning of thermodynamics

$$\left( \frac{1 - \varphi_a}{\varphi_a} \right) \cdot \left( \frac{C_6^2}{2} \right) = h(T_5) - h(T_1) + \left( \frac{\Omega}{j} \right) \cdot (p_2^j - p_1^j), \quad (6)$$

where  $j$  – the degree indicator, which is related to the indicator of the polytropic process by the ratio

$$j = \frac{n-1}{n}, \quad (7)$$

where  $\Omega$  is a constant in the polytropic process equation, which is determined by the expression

$$\Omega = v_6 \cdot \sqrt[n]{p_6}. \quad (8)$$

The specific volume of steam at point "5" is determined by the equation of the polytropic process of gas flow in the nozzle with friction

$$v_5 = \frac{\Omega}{\sqrt[n]{p_1}}. \quad (9)$$

According to equation (4), we determine the temperature of the steam at the point "5" at the entrance to the nozzle apparatus, which is necessary for the continuity of the parameters of the working body

$$v_5 = \left[ 0.6 \cdot (T_5 - 303)^{0.5} + 2.898 \right] \cdot (10^{-3}). \quad (10)$$

Equations (5) – (10) determine the parameters of R-744 gas movement in the turbine nozzle apparatus. To solve the system (5) – (10) of nonlinear algebraic equations, it is advisable to apply the method Newton [5]. As an example for  $p_2 = 72.137$  MPa (initial pressure at the nozzle entrance) we give the following solution  $T_5 = 321$ .K (48.9°C),  $v_5 = 5.51 \cdot 10^{-3}$  m<sup>3</sup>/kg,  $C_6 = 193$  m/s,  $n = 1.12$ ;  $C_0 = 17$  m/s. The parameters of the R-744 gas phase at point "6" in figure 1 were taken from the reference tables at  $T_1 = 278$ K (5°C). On line "1-6" of figure 1, the temperature is constant and equal to  $T_1$ . For well-profiled blades of the nozzle device, the speed coefficient can be taken to be of the order of 0.95.

Let's consider the issue of choosing an effective mode of operation of the turbine based on the level of thermodynamic efficiency. Based on general thermodynamic laws [6], the expression for cycle efficiency is the ratio of useful mechanical work to the heat given off by the working body (carbon dioxide) in a condenser cooled by water from an external source

$$\eta_{744}(T_6) = \frac{(P_{c744}(T_5) - P_{c744}(T_6))}{L_{744}(T_6)} \cdot \left( v_5 - v_{sL744}(T_1) + \int_{v_5}^{v_6} p_{5,6}(v) \cdot dv \right), \quad (11)$$

where  $P_{c744}(T)$  – the pressure of saturated R-744 vapor (carbon dioxide) depending on temperature, MPa;  $L_{744}(T)$  – specific heat of condensation of R-744 saturated vapor depending on temperature J/kg;  $v_{sL744}(T)$  – specific volume of saturated R-744 liquid depending on temperature, m<sup>3</sup>/kg,  $p_{5,6}(v)$  – pressure change function depending on specific volume during expansion of R-744 steam in the turbine, Pa.

The function  $p_{5,6}(v)$  is found by the method of effective indicators of the isentropic process [7]. To model the properties of R-744 (carbon dioxide), we use the Redlich-Kwong equation [8]. The integral on the right-hand side of (1) determines the useful mechanical work on the turbine blades. The function of the saturated vapor pressure of R-744 freon depending on the temperature is determined by the expression

$$P_{sCO_2}(\tau) = P_{c744} \cdot \left( 1 - 6.991754 \cdot \tau + 24.8393 \cdot \tau^{1.91} - 16.70606 \cdot \tau^{2.12} \right), \quad (12)$$

where  $P_{c744}$  – is the pressure at the critical point of R-744, which is equal to 7.386 MPa;  $\tau$  – is the relative temperature determined by the expression

$$\tau = \frac{(T_{c744} - T)}{T_{c744}}, \quad (13)$$

where  $T_{c744}$  – is the temperature at the critical point of R-744, which has a value of 304.33 K.

To shorten entries in (12), the R-744 saturated vapor pressure function is presented depending on criterion (13). The value of specific heat  $L_{744}$  of R-744 vapor depending on temperature according to experimental data [2] is represented by expression (3). The diagram for the Rankine cycle with carbon dioxide for our temperature range is close to the critical point, and therefore accounting (taking into account) the specific volume of saturated liquid carbon dioxide is essential in the calculations of the efficiency of the cycle. To construct the desired function  $v_{sL744}(T)$ , we use the experimental data on the

specific volume of the saturated liquid phase R-744, presented in the reference book [2]. These data are shown in table 1.

**Table 1.** Experimental data on the specific volume of R-744 depending on the temperature and the results of its mathematical modeling.

| $t, ^\circ\text{C}$ | $T, \text{K}$ | $v_{e744},$<br>0.001 m <sup>3</sup> /kg | $v_{sL744}(T),$<br>0.001m <sup>3</sup> /kg | $r,$<br>relative error, % |
|---------------------|---------------|---|--|---------------------------|
| 31.03               | 304.03        | 2.1367                                  | 2.1367                                     | 0                         |
| 30                  | 303           | 1.685                                   | 1.671                                      | 0.84                      |
| 25                  | 298           | 1.407                                   | 1.397                                      | 0.83                      |
| 20                  | 293           | 1.293                                   | 1.282                                      | 0.85                      |
| 15                  | 288           | 1.218                                   | 1.282                                      | 0.84                      |
| 10                  | 283           | 1.161                                   | 1.154                                      | 0.59                      |
| 5                   | 278           | 1.116                                   | 1.112                                      | 0.47                      |
| 0                   | 273           | 1.078                                   | 1.074                                      | 0.32                      |

The function of the density of the saturated liquid phase of R-744 depending on the temperature is determined by the expression based on the scale theory [9] of the critical point for working substances

$$\rho_{sL744}(\tau) = \rho_{c744} + 123.625 \cdot (T_{c744} - T)^{0.391377} + 7.030055 \cdot (10^{-2}) \cdot (T_{c744} - T)^2 - 0.616157 \cdot (T_{c744} - T), \quad (14)$$

where  $\rho_{c744}$  – is the density of R-744 at the critical point, 467 kg/m<sup>3</sup>;  $T_{c744}$  – is the temperature of R-744 at the critical point, 304.03 K.

Then the specific volume is determined by the expression

$$v_{sL744}(T) = 1/\rho_{sL744}(T). \quad (15)$$

The results of calculations based on expression (14) are presented in table 1 and compared by relative error

$$r = \frac{|v_{e744} - v_{sL744}(T)|}{v_{e744}} \cdot 100 \quad (16)$$

with reference data [2]. The accuracy of the mathematical modelling of the specific volume of the saturated liquid phase R-744 depending on the temperature according to the relative error is better than 1 percent.

Expression (11) for the function  $v_{sL744}(T)$  is given by (14). When calculating the efficiency according to (11), the numbering and location of the key points of the Rankine thermodynamic cycle for R-744 in "p-v" coordinates is given in figure 1. To substitute in expression (11), it is necessary to find  $T_5$ ,  $v_5$  and the function  $p_{5,6}(v)$ . Calculations are carried out from point "6", the parameters of which are set according to the conditions of the cycle organization. They are determined by the temperature of the available cooling medium in the condenser. We aim to investigate the reduction of this temperature to increase the efficiency of the cycle. We determine  $T_5$  and  $v_5$  using the Redlich-Kwong equation, which for point "5" has the following form

$$p_2 = \left( \frac{R_{744} \cdot T_5}{v_5 - b_{744}} - \frac{a_{744}}{v_5 \cdot (v_5 + b_{744}) \cdot \sqrt{T_5}} \right), \quad (17)$$

where  $R_{744}$  – the gas constant for the working fluid R-744, which is equal to 188.95 J/(kg·K),  $T_5$  – the temperature of R-744 at point "5", K;  $v_5$  – specific volume of R-744 at point "5", m<sup>3</sup>/kg;

$b_{744}$  – a constant that characterizes the forces of intermolecular repulsion for R-744 and is equal to  $6.675 \cdot (10^{-4})$  ( $\text{m}^3/\text{kg}$ );  $a_{744}$  – a constant that characterizes the forces of intermolecular attraction, and for R-744 it has a value of  $3331.2$  ( $\text{N} \cdot \text{m}^4 \cdot \text{K}^{0.5} / \text{kg}^2$ ).

We also use the equation of the first law of thermodynamics, which is expressed in terms of enthalpy. The use of enthalpy is convenient because its reserve determines the gas velocity in the turbine nozzles. We integrate the differential equation of the first law of thermodynamics using enthalpy along line "5-6" in figure 1

$$\int_{T_1}^{T_5} C_{v744}^0(t) \cdot dt + (p_1 \cdot v_6 - p_2 \cdot v_5) = v_6 \cdot \int_{p_1}^{p_2} \sqrt{\frac{p_1}{p}} \cdot dp + \left( \frac{3 \cdot a_{744}}{2 \cdot b_{744}} \right) \cdot \int_{T_1}^{T_5} t^{-1.5} \cdot \ln \left[ 1 + \left( \frac{b_{744}}{v_6} \right) \cdot \left( \frac{T_1}{t} \right)^{\frac{1}{n_t}} \right] \cdot dt, \quad (18)$$

where  $C_{v744}^0(v, T)$  – isochoric heat capacity of R-744 in the ideal gas state [8]

$$C_v^0(v, T) = 1003 + 0.205 \cdot T + 1.941 \cdot 10^{-7} \cdot T^{-2} \quad (19)$$

where  $n$  and  $n_t$  are the effective indicators of the polytropic process "5-6" in the " $p$ - $v$ " coordinates and in the " $v$ - $T$ " coordinates, respectively.

The effective indicators of the isentropic process "5-6" in the coordinates " $p$ - $v$ " and in the coordinates " $v$ - $T$ " are found, respectively, from the equations

$$\ln \left( \frac{p_1}{p_5} \right) = n \cdot \ln \left( \frac{v_5}{v_6} \right), \quad (20)$$

$$\ln \left( \frac{T_1}{T_5} \right) = n_t \cdot \ln \left( \frac{v_5}{v_6} \right). \quad (21)$$

The system of equations (17) – (21) is closed with respect to the number of unknowns and can be solved by numerical methods. For example, by Newton's method [4].

Calculations show that at  $T_5 = 29^\circ\text{C}$  and  $T_1 = 20^\circ\text{C}$  the value of the efficiency of the Rankine cycle on R-744 is 10.4%, at  $T_5 = 29^\circ\text{C}$  and  $T_1 = 5^\circ\text{C}$  the value of the efficiency of the cycle is 23%. Therefore, the development of technical solutions to reduce the cooling of the R-744 steam condenser after the turbine is a promising direction for the use of secondary heat resources of  $30^\circ\text{C} - 8^\circ\text{C}$ . The additional electricity generated by the  $\text{CO}_2$  turbine can support the operation of the equipment to process the accumulated waste of the mining industry [10].

#### 4. Conclusions

Analytical expressions for the thermophysical parameters of  $\text{CO}_2$  for temperatures of  $3-80^\circ\text{C}$  were obtained on the basis of mathematical modelling methods, and effective modes of the gas turbine for the use of secondary heat resources of mining enterprises were investigated based on them.

A study of the influence of the cooling temperature of the working body on the efficiency of the Rankine cycle was carried out on the basis of the appropriate mathematical model. Calculations show that at  $T_5 = 29^\circ\text{C}$  and  $T_1 = 20^\circ\text{C}$  the value of the efficiency of the Rankine cycle on freon R-744 is 10.4%, at  $T_5 = 29^\circ\text{C}$  and  $T_1 = 5^\circ\text{C}$  the value of the efficiency of the cycle reaches 23%. Ensuring such condenser cooling temperatures is possible when using mine water. Therefore, developing technical solutions to reduce the cooling of the R-744 steam condenser after the turbine is a relevant and effective way of using secondary heat resources in the temperature range of  $30^\circ\text{C} - 8^\circ\text{C}$ , the volumes of which are pretty significant in mining enterprises.

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