Nonlinear Analysis of Bifurcatory Properties of Mathematical Model of Subpopulation Dynamics in the Case of a Single Niche for Subpopulation

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Abstract —**The article is devoted to the use of mathematical models of the dynamics of heterogeneous populations, and computer simulation based on the above models allows to identify general trends in subpopulations, predict the state of the system and obtain results on possible consequences of artificial intervention. Also, the use of mathematical models can predict the spread of genetic anomalies. The authors propose a model of subpopulation dynamics with a logistic function as a basic one. It is concluded that the system-wide dynamics of subpopulation processes depends not only on the reproductive potential of subpopulations, but also on the intrasystemic dynamics that objectively occur in such systems. The adequacy of the proposed mathematical model is proved.**

Keywords—Mathematical Model, Computer-Based Simulation, Phase Portrait, Phase Trajectory, Bifurcation Analysis, Bifurcation Properties of the System

I. INTRODUCTION

Anthropogenic pressure leads not only to a global deterioration of the environment, but also to an increase in the number of pathologies of biological objects (decreased immunity, impaired reproductive function, etc.) [1], [2]. In the conditions of instability of environmental factors, mosaic of habitats of species the genetic heterogeneity of species and separate populations considerably increases [3]. The peculiarities of this situation should be taken into account when planning environmental measures, conducting environmental monitoring, solving problems of forecasting the future state of populations [4] - [6]. It is extremely important to study the level of genetic heterogeneity of the human population. The accumulation of pathological recessive genes can be hidden for a long time, and at some point manifest itself in the form of a rapid increase in cases of certain inherited diseases.

The use of mathematical models of the dynamics of heterogeneous populations, computer simulation based on them allows you to effectively, with minimal time and material resources to identify general trends in subpopulations, predict the state of the system and possible consequences of artificial intervention [7] - [9]. In particular, mathematical modeling can predict the spread of genetic abnormalities [10].

In the classical form the system of dynamics of conditionally isolated populations is presented in the form $\dot{x} = f_i(\overline{x}), i = \overline{1,n}$, where x_i – the number of the i^{th} population, $f_i(\overline{x})$ – determines its reproductive capabilities. The authors propose to consider the model of subpopulation dynamics in the form $\dot{x}_i = \sum_{j=1}^{n} A_{ij} f_j(\overline{x}), i = \overline{1,n}$, that is, when the function $f_j(\overline{x})$, $j = \overline{1,n}$ is included in the right-hand side of each equation with some $A_{ij} \in [0;1]$ coefficient, which will be called the transition coefficient.

II. MATHEMATICAL MODEL OF SUBPOPULATION DYNAMICS WITH LOGISTIC FUNCTION AS BASIC

The study is based on the idea of the population as a set of individuals, which can be divided into n subpopulations that are genetically more or less homogeneous, but significantly different from each other. They are not reproductively isolated, and there is a certain probability of offspring from the *i-th* subpopulation to the *j-th* subpopulation. The differential model of the system can be written in general as follows:

$$\frac{dx_j}{dt} = \sum_{i=1}^{n} A_{ji} \cdot f_i(x), j = \overline{1,n}, \quad (1)$$

where x_j is the number of the j-th subpopulation, and $f_i(x)$ is a function that describes the total reproductive capabilities of the i-th subpopulation, and A_{ji} is the share of the descendants of the i-th subpopulation, which falls into the j-th.