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## THE ACCURACY OF TRACTION CURRENT HARMONICS PARAMETERS DETERMINATION BY WINDOWED FFT

### 1. Introduction

The problem of electromagnetic compatibility (EMC) between railway subsystems has attracted the attention of scientists in recent years due to the wide spread of high-speed railways [1-4]. The one of the main sources of the electromagnetic disturbances in electrified railways is the rolling stock and the traction system as a whole. New types of rolling stock should be tested on EMC with railway signaling systems before they are allowed to operate [5,6]. Also monitoring of electromagnetic interference in signaling circuits should be periodically performed during the entire period of their operation in accordance with the schedule [7-11]. The maximum permissible values of electromagnetic interference, which can be produced by the electrical equipment of the rolling stock, are described in standards and norms [12-14]. The admissible values of the harmonics of the traction current in rails are presented in table 1 [14].

Methods for measuring electromagnetic interference (EMI) in railway lines were discussed in [5,6]. The RMS EMI values of the traction current (TC) in the frequency ranges specified by the standards can be measured by filtering the traction current using a bank of parallel filters with specified frequency bands or by a fast Fourier transform (FFT).

The amplitude and frequency of the traction current harmonics, in general, vary with time and therefore the FFT in classical form is not able to determine accurately the spectral characteristics of the traction current. For spectral analysis of non-stationary signals Dennis Gabor [15] proposed the Short-Time Fourier transform (STFT) that applies FFT to signal divided into segments of certain length

(by sliding windows) where the signal could be admitted as stationary one.

Since electromagnetic interference in signal lines, in particular in rail lines, is capable to cause dangerous disruptions in the train control systems operation, strict requirements are applied to the accuracy of interference measurement technique [14].

The STFT spectral analyzing techniques have some constraints, and a one of them is connected to trade-off between the FFT window length and the frequency band.

According to the uncertainty principle [16] the product of the time and frequency resolutions is lower bounded by

$$\Delta t \Delta f \geq \frac{1}{4\pi},$$

where  $\Delta t$  and  $\Delta f$  denote the time and frequency resolutions, respectively.

Another constraint of STFT is related to the side-lobe levels of the window in the frequency domain, which leads to a decrease in the measured values of the RMS current of the harmonics and in some cases can even to hide the weak harmonics against the background of strong harmonics [17-21]. To improve the accuracy of the spectral analysis of the traction current, it is necessary to correctly choose the parameters of the windowed FFT.

### 2. Aim of the article

The aim of the article is to give a brief overview of choosing the correct parameters of the windowed FFT to achieve the necessary time and frequency resolution of the spectral analysis of traction current in accordance with the requirements of regulatory documents and to investigate the influence of FFT parameters on accuracy of determining the main parame-

ters (such as effective current, frequency, duration) of the harmonics in the traction current spectrum.

This paper is organized as follows. Section 3 describes the main features of the TC harmonics technique, section 4 – the parameters of the windowed FFT, section 5 – choosing of window parameters, and section 6 – results of simulation. Section 7 concludes the work.

### 3. The main features of the measurements of TC harmonics

Dynamic range of the traction current harmonics is defined as ratio of the main harmonic level (for AC TC it is 50 Hz harmonic) to the level of the weakest harmonic. But it is interesting to find this ratio for all harmonics specified by standard [14], i.e. e. to calculate values of dynamic range for every harmonic with rated frequency table 1 as

$$D_f = 20 \log_{10} \left( \frac{I_o}{I_f} \right), (in \text{ dB}).$$

where  $I_o$  and  $I_f$  are the root-mean square values of main harmonic and harmonic with rated frequency  $f$ .

As a result, the dynamic range for AC TC with RMS current 200 A is 46 dB for frequency band 21-29 Hz, 55 dB for frequency bands 408–432, 468–492, 568–592, 708–732, 768–792 Hz, and 60 dB for frequency bands 4462–4538, 4962–5038, 5462–5538 Hz. It is obvious that the dynamic range of harmonics in these frequency bands reaches higher values when the traction current gets higher values.

The obtained values of the dynamic range have sufficiently high values, and they should be taken into account when selecting the window type.

The requirement that the duration of the harmonics defined by the standard [14] cannot exceed 0.3 s, together with the restriction on the values of the frequency and time resolutions, should also be taken into account when selecting the window type, especially in the frequency range from 19 Hz to 58 Hz, which

contains very closely spaced harmonics, which are specified by the standard (table 1).

**Data acquisition technique.** According to the Nyquist criterion, the sampling frequency  $F_s$  must be at least twice the maximum frequency of the harmonic of interest. Before digitizing the signal, an antialiasing filter must be used to reduce the frequency components at and above half the sampling frequency to a level below the dynamic range of the analog-to-digital converter (ADC). In accordance with the dynamic range, calculated above for the traction current of 200 A, at least 10 bits of the ADC must be used to digitize the TC. For higher values of traction current, a 12-bit A/D converter must be used.

Table 1.

**The permissible effective values of the harmonic currents produced by the rail car's electrical equipment**

Electrical supply system	Frequency band, Hz	Rated frequency, Hz	Permissible RMS current, A
DC, 3 kV	19-21	25	11,6
	21-29		1,0
	29-31		11,6
	40-46	50	5,0
	46-54		1,3
	54-60		5,0
	4507—4583	4545	0.2
	5517—5593	5555	0.2
AC 50 Hz, 25 kV	15-21	25	4,1
	21-29		1,0
	29-35		4,1
	65-85	75	4,1
	4462-4538	4500	0,2
	5462-5538	5500	0,2
DC, 3 kV and AC 50 Hz, 25 kV	167-184	175	0,4
	408-432	420	0,35
	468-492	480	0,35
	568-592	580	0,35
	708-732	720	0,35
	768-792	780	0,35
	4962-5038	5000	0,2

#### 4. The windowed FFT parameters

For each specific time  $t$ , a short time Fourier transform is performed on the signal  $x(t)$ , using a sliding window function  $w(t)$  centered at time  $t$ . Consequently, the window is moved by  $\Delta t$  along the time line, and another Fourier transform is performed [17-19]. Therefore, the STFT is performed by shifting a window of constant size along the time axis of a signal, so that the signal is divided into segments in which it is considered stationary and then, the FFT can be applied. As a result the various frequency components at different times can be resolved for non-stationary signal.

The usual mathematical definition of the STFT for sampled signal is [18-21]

$$X_m(\omega) = \sum_{n=-\infty}^{\infty} x(n)w(n-mR)e^{-j\omega n},$$

where  $x(n)$  is input signal at time  $n$ ,  $w(n)$  is window function of length  $M$ ,  $X_m(\omega)$  is FFT of windowed data centered about time  $mR$ ,  $R$  is hop size, in samples, between successive FFTs.

Over the past few decades, various types of window functions have been proposed, and the correct choice of window type and its parameters according to the signal to be analyzed is an important task for obtaining the correct analysis results.

#### 5. Characteristics of windows

The windows are used to reduce spectral leakage when performing a Fourier Transform on time but they cannot eliminate leakage entirely, and they only change the shape of the leakage. Each type of window affects the spectrum in a slightly different way. For each specific application the best type of window should be chosen suited to it.

The FFT window is characterized by certain parameters that must be taken into account for the correct choosing of window, such as [17-20]: half main lobe width  $B$ , amplitude correction factor  $\beta$ , the maximum side-lobe level  $\gamma_{\max}$  and side lobe roll-off. The last one

is the asymptotic decay rate of the peaks of the side lobes, expressed in decibels per decade of frequency.

Choosing the windows type and parameters should be based on the specific application. To spectral analysis of traction current the high amplitude, time and frequency resolution windows type are necessary.

The dynamic range of FFT window is defined as the ratio of main-lobe amplitude to first side-lobe amplitude in frequency domain and should be chosen greater than dynamic range of traction current that analyzed.

The window itself distorts both the amplitude and energy of the signal and hence it results in amplitude and frequency distortion of harmonics. To compensate the effects of applying a window to data the window correction factors must be used. Their values are determined by the window type that was applied.

The correct timing and frequency resolution of the STFT is necessary to allow for a trade-off between them. The time resolution  $\Delta t$  of the window function can be presented analytically as [22]

$$\Delta t^2 = \frac{\int t^2 |w(t)|^2 dt}{\int |w(t)|^2 dt}.$$

Similarly, the frequency resolution  $\Delta f$  is defined as [23]

$$\Delta f^2 = \frac{\int f^2 |W(f)|^2 df}{\int |W(f)|^2 df},$$

where  $W(f)$  is the Fourier transform of the window function  $w(t)$ .

The frequency resolution capability  $\Delta\omega_w$  for certain window depends of window main-lobe width  $B_w$

$$\Delta\omega_w = \frac{B_w F_s}{N_w},$$

where  $F_s$  is the sampling frequency and  $N_w$  is the window size (in number of samples).

Two harmonics can be separated in spectrum if their frequency difference  $\Delta\omega_s$  is greater than frequency resolution capability  $\Delta\omega_w$ .

$$\Delta\omega_s > \Delta\omega_w.$$

Resolution in time depends of time duration of window determined by

$$T_w = \frac{N_w}{F_s}.$$

For STFT with overlapping windows the sample series of length  $x$  is divided on  $k$  segments, where  $k$  defined as

$$k = \frac{N_x - Nov}{N_w - Nov}.$$

where  $N_x$  is the length of  $x$ -sample series,  $Nov$  is a number of overlapping samples.

## 6. Simulation results

To evaluate an accuracy of determining the root-mean-square values of the TC harmonics, their durations and frequency bands by using windowed FFT, four different types of high- and moderate-resolution windows have been chosen (table 2).

A rectangular window (or "non-window", or uniform window) has a value of 1.0 across the entire measurement time and is the simplest window that cuts a segment of the signal without any other modification at all, which leads to discontinuities at the ends. It was chosen in the work for carrying of comparison with the results obtained for other windows. The main lobe is narrow, but the side lobes are very large and roll off quite slowly. The first side-lobe is only 13 dB lower than the main lobe, with the rest side lobes falling off at about 6 dB per octave [18]. The main lobe is quite rounded and can introduce large measurement errors. The rectangular window can have amplitude errors as large as 36%.

Hann (or Hanning) windows are often used with random data because they have moderate impact on the frequency resolution and amplitude accuracy of the resulting frequency spec-

trum, especially when compared to the effects of other windows. The highest side lobe is 32 dB lower than the main lobe. The rest side lobes falling off at about 60 dB/octave. This window is most useful for searching operations where good frequency resolution is needed, but amplitude accuracy is not important. The Hanning window will have amplitude errors as much as 16%.

The Hamming window has the lowest possible side-lobe level among all windows based on three Dirichlet kernels. The highest side lobe is 48 dB lower than the main lobe. The rest side lobes falling off at about 6 dB/octave.

The Blackman window uses five Dirichlet kernels, thus reducing the side-lobe level still further. The Blackman window offers good side lobe attenuation, near to -58 dB and the side lobes roll off at approximately 18 dB per octave. Therefore, the Blackman window should be chosen if the signal contains strong interference near the frequency of interest. On the other hand, the main lobe width in the Blackman window is wider than in the Hamming one, so, if the spectral resolution is crucial, the best choice would be the Hamming window.

Table 2.

Parameters of the windows

Window type	$B$	$\gamma_{max}$ , dB	$\beta$ , dB
Rectangular	2	-13	0
Hann	4	-31.5	-6
Hamming	4	-42	-5.37
Blackman	6	-58	-7.54

For the windowed FFT spectral analysis, an analytically synthesized signal was used with known amplitude and frequency parameters of the harmonics, the values of which were taken in accordance with the permissible parameters of the harmonics (table 2). Knowledge of the signal parameters makes it possible to estimate the accuracy of the spectral analysis of the traction current using a window FFT with the chosen parameters.

The main frequency of the signal was taken as 50 Hz and RMS value – 200 A. RMS values and frequencies of other harmonics in

the synthesized signal were corresponded to the permissible RMS current values and rated frequencies presented in table 1. For convenience RMS values for harmonics in synthesized signal were recalculated into relative units in order that effective value of the main harmonic to be equal unity ( $I_{50}=1$ ). Parameters of the harmonics are presented in table 3, column 1, 2. The sampling frequency for the signal was chosen as 27500 Hz according to the Nyquist frequency requirements, and it is better if it be chosen with a certain margin like.  $F_s = 5 \cdot f_{\max}$ . Lengths of windows were taken as 32768 samples according to requirement that maximum permitted time for disturbances is 0.3 s [14]. Frequency resolution can be calculated as 0.27 Hz for rectangular window, 0.54 Hz for Hann and Hamming windows, and 0.81 Hz for Blackman window. Such values of frequency resolution are satisfactory for TC spectral analysis.

Spectrums of the synthesized signal obtained by windowed FFT with windows length 0.3 s are shown in fig. 1. Obtained RMS current values and frequencies of the harmonics are marked in fig. 1. All peaks in spectrum are smeared due to spectrum leakage and its frequency resolution is minimal for rectangular window and maximal for Blackman window because last one has the lowest side-lobe level -58 dB.

The ratio errors of the measured frequencies and RMS values of the harmonics by windowed FFT are presented in table 3. Frequency resolution of the harmonics increased and, correspondingly the ratio error of frequency decreased with increasing frequency. The type of windows used slightly affects the frequency resolution of the TC harmonics. The values of the ratio error of the RMS harmonic currents are higher for rectangular window compared to Hann, Hamming and Blackman windows, and ratio error values are decreasing in the sequence from rectangular to the Blackman window, but even for the last one the accuracy for some frequencies does not satisfy the requirements of practical application. The high values of ratio error are explained by the spectrum leakage and scalloping.

Due to the inverse proportional relationship between the signal frequency and the time resolution, an increase in the frequency resolution can be achieved for windows of greater length. Therefore, the computer investigation of the windowed FFT accuracy has been carried out for windows of length 1 s.

In this case, the resolution of the TC harmonics was better (fig. 2), and the ratio error of the root-mean-square current and harmonic frequencies is much lower than for windows with a length of 0.3 s (table 2). But even in this case the accuracy for some frequencies does not satisfy the requirements of practical application.

To ensure the necessary accuracy of the traction current spectral analysis with using windowed FFT, proper choosing of the spectral analysis parameters taking into account the traction current parameters is necessary. Taking into account a wide frequency range of traction current, a technique with variable window length and sampling frequency for different parts of the spectrum is perspective for using.

## Conclusion

In the work the correct choosing of the windowed FFT parameters to achieve the necessary time and frequency resolution of the traction current spectral analysis in accordance with the requirements of regulatory documents has been briefly overviewed and investigations of the influence of FFT parameters on the accuracy of the determination of harmonics parameters (such as effective current, frequency, duration) have been carried out.

To assess the accuracy of determining the RMS current and frequency of the harmonics, a computer study was performed using a synthesized current with known harmonics parameters, the values of which were chosen in accordance with the permissible values of the parameters determined by regulatory documents and standards.

For the spectral analysis of traction current, four types of windows were selected: rectangular, Hann, Hamming and Blackman windows with duration of 0.3 and 1 s. For a sam-

pling frequency of 27500 Hz and a window length of 0.3 s, the frequency resolution is 0.27 Hz for a rectangular window, 0.54 Hz for Hann and Hamming windows and 0.81 Hz for Blackman's window, which is consistent with the requirements of the specifications.

The results of spectral analysis of traction current showed that the frequency resolution of harmonics, and accordingly the relative error in determining the frequency and the RMS value of harmonics is lower for high frequencies in spectrum.

The type of windows used has a slight effect on the accuracy of determining the frequency of harmonics. The relative error of the effective value of the harmonic current was

higher for a rectangular window, and relative error decreased in the row from the rectangular window to the Blackman window. The values of the relative error of the RMS current for several frequencies of the harmonics did not meet the requirements necessary for the practical use of the method, and this is due to spectrum leakage and scalloping.

For windows with a length of 1 s, the frequency resolution of the traction current was higher. than for windows with a length of 0.3 s, and the relative error of the RMS current and frequencies of the harmonics were much lower, but even in this case, the relative error was high for individual frequencies.

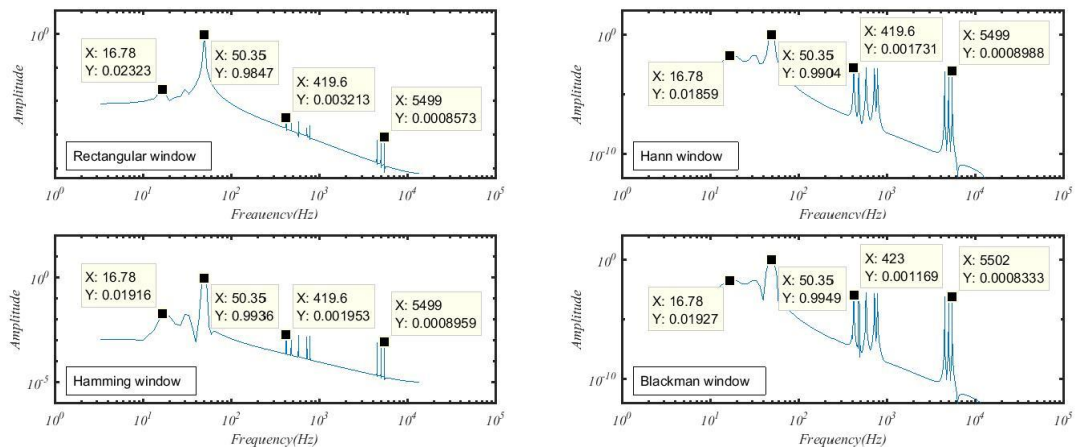


Fig. 1. Signal spectra for rectangular, Hann, Hamming, and Blackman windows length 0.3 s.

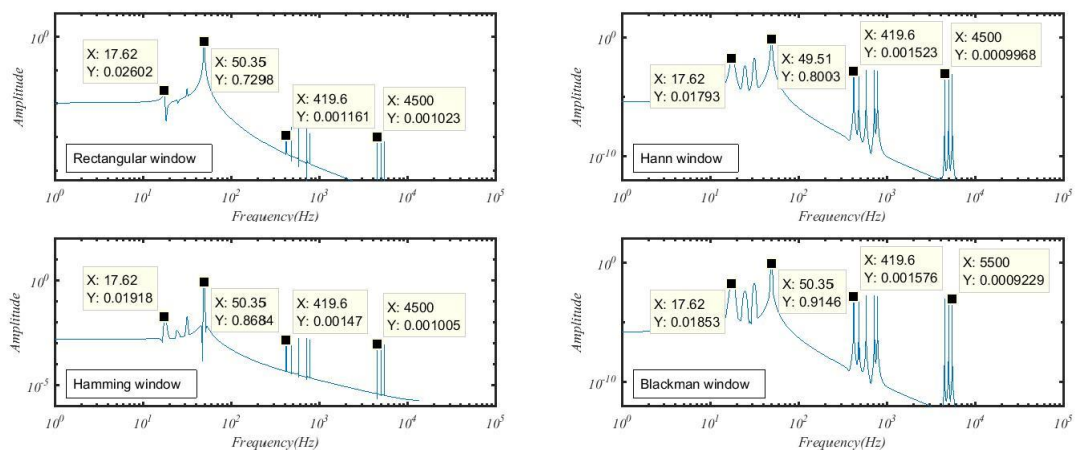


Fig. 2. Signal spectra for rectangular, Hann, Hamming, and Blackman windows length 1 s.

Table 3

**Measurement results**

f, Hz	I*10 <sup>3</sup> , relative units	The ratio error of frequency and amplitude measurements									
		Window length 0.3 s					Window length 1.0 s				
		$\delta f$ , %	$\delta I$ , %				$\delta f$ , %	$\delta I$ , %			
			RW	HnW	HmW	BW		RW	HnW	HmW	BW
18	20.50	6.75	13.3	9.32	6.54	6.01	2.09	26.9	12.5	6.42	14.2
25	5.00	6.00	167	38.3	26.2	101.1	0.71	172.2	3.02	9.49	2.13
32	20.50	5.58	12.3	12.3	12.7	7.29	0.34	41.9	1.34	2.50	0.80
50	1000.00	0.71	1.53	0.96	0.64	0.51	0.97	27.0	19.9	13.2	8.54
420	1.75	0.09	83.6	1.08	11.6	0.61	0.09	34.5	12.9	15.9	9.95
480	1.75	0.01	76.3	0.25	11.5	0.03	0.01	15.0	0.39	2.29	0.06
580	1.75	0.12	45.3	3.41	3.42	2.43	0.02	8.59	0.97	0.82	0.51
720	1.75	0.22	10.4	14.3	15.5	12.7	0.01	8.94	0.59	1.22	0.22
780	1.75	0.15	10.6	8.08	5.95	6.10	0.06	25.4	20.1	12.9	8.45
4500	1.00	0.04	29.1	14.8	18.5	12.3	0.001	2.34	0.32	0.47	0.01
5000	1.00	0.03	21.2	12.8	13.8	14.0	0.003	4.24	2.34	2.23	1.59
5500	1.00	0.02	14.26	10.12	10.41	16.66	0.01	24.90	21.35	11.94	7.70

Designations: RW – rectangular window; HnW – Hann window, HmW – Hamming window; BW – Blackman window.

To ensure the necessary accuracy of the traction current spectral analysis with using windowed FFT, proper choosing of the spectral analysis parameters taking into account the traction current parameters is necessary. Taking into account a wide frequency range of traction current, a technique with variable window length and sampling frequency for different parts of the spectrum is perspective for using.

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- Ключові слова:** спектральний аналіз, тяговий струм, коротке часове перетворення Фур'є, електромагнітна сумісність..
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