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Development of a Spectral Theory for Analysis of Non-Stationary Pulse Stochastic Electromagnetic Processes in Devices of Electric Transport Systems

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***Abstract* — A spectral analysis of traction voltages and currents is a basis for estimation of electromagnetic compatibility level and quality of consumed power in electric transport systems. However, such an analysis is usually performed for steady-state modes and only under the condition that the time realization of voltages and currents, being deterministic for continuous quantities, have infinite length. De facto, the electric transport devices operate in non-stationary dynamic modes (starting up, coasting, acceleration, regenerative braking, stopping, wheel spin, voltage surges, etc.). As a result, the voltage across the traction motors and the current flowing through them are noncontinuous pulsed stochastic processes. It is necessary to add that in emergency modes the voltage and current are short-term single pulses. The paper presents the spectral analysis of such random sequences of pulses as well as their fronts and decays, the concepts of actual and instantaneous spectra. The analytical expressions for amplitudes and the initial phases of k-th harmonics are obtained using the discrete Fourier transformation. The numerical calculations of the spectral composition of stochastic pulse processes of voltage and current were performed for the DE1 and VL8 electric locomotives (Ukraine) as well as for trams operating on the routes of the city of Dnipro. The actual and instantaneous spectra, as well as the spectra of the full correlation functions and their "tails", were determined for the electric traction voltages and currents.**

Keywords* — *spectrum, electric transport, voltage, pulse, current, stochastic, instantaneous, correlation function.

I. Introduction

Currently, the spectral composition of traction voltages $U(t)$ and currents $I(t)$, the quality indices of electric power and the level of electromagnetic compatibility of devices in the electric transport systems are defined; first, in steady-state modes and, second, with the condition that $U(t)$ and $I(t)$ realizations have infinite duration [1]-[5]. Also, the spectra of deterministic non-sinusoidal voltages and currents are mainly investigated. In particular, the noncanonical harmonic composition of non-sinusoidal periodic voltages and currents in the external traction power supply lines have been established by [6]-[8] using the classical Fourier analysis. The discrete spectral composition of the output voltage of traction substations equipped with 6- and 12-pulse diode rectifiers were investigated with the use of the same method in [9]. The harmonic composition of low-frequency pulsations of the output voltage of 6-pulse controlled (thyristor) rectifiers were thoroughly analysed in [10]. Harmonic spectra of traction current of locomotives with a thyristor converter and locomotive with resistor control was compared in [11], [12]; the authors also found the harmonics created by traction substation in the currents' variable component of the locomotive, rail circuits and return wire. Researches [13], [14] constitute exceptions in which the voltage and the traction-recuperation current are considered as continuous, endless, stationary random processes, so their spectra $S(\omega)$ were defined by the formula of the direct Fourier transform as the spectrum of one realization with infinite duration $f(t)$ of a random process of voltage or current:

$$S(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt \,. \qquad (1)$$

However, electric transport devices (traction substations, traction networks, electric rolling stock, etc.) operate in non-stationary, dynamic modes: starting up of an electric rolling stock (ERS), its acceleration, coasting, braking, a sharp increase of the load on the high grades, regenerative braking, various kinds of short circuits, wheels spin and slip, operational interruptions and voltage surges. As a result of frequent detachments (with subsequent restoration) of the current collector from the contact wire, the voltage and current are interrupted. Experimental studies showed that even after detachment for more than 0.05 s the current of vehicle tends to zero, while the actual durations reach up to

1.5 s. In addition, several factors are responsible for the appearance of voltage surges and dips on the current collector. The first one with the value of $(0.07…0.33)\cdot U_{nom}$ is observed with a probability of 0.3…0.8, and the second has the same value, but its probability is 0.36…0.7. Also the influence of transient modes on the distortion of the form of the traction current should be pointed out: at operating modes of 0.5…1.5 s the current surges reach $(1.5…1.7)\cdot I_{nom}$ and steepness of 50…1300 A/s; in emergency modes with duration of 0.05…0.3 s, respectively current surges are up to $(5…6)\cdot I_{nom}$ and steepness 10…200 kA/s.

In the mentioned above operational modes, the voltage applied to the traction motors (separated from voltage of the locomotive on the yellow zone in Fig. 1a) and the current flowing through them are interrupted, in particular, they have certain values in the traction mode, in coasting mode they take zero value and in a regenerative mode - negative values.

In non-stationary emergency modes, the voltages and currents (switched off by the protection equipment) always represent short-term single pulses (Fig. 2).

Consequently, in any mode the time dependencies of electric traction voltages and currents in electric transport systems are random sequences of short-term non-periodic pulses of arbitrary shape [15], [16]. *In this connection, the aim of the work is the development of the theory of spectra for the analysis of non-stationary pulse stochastic electro-magnetic processes in DC electric transport systems.*

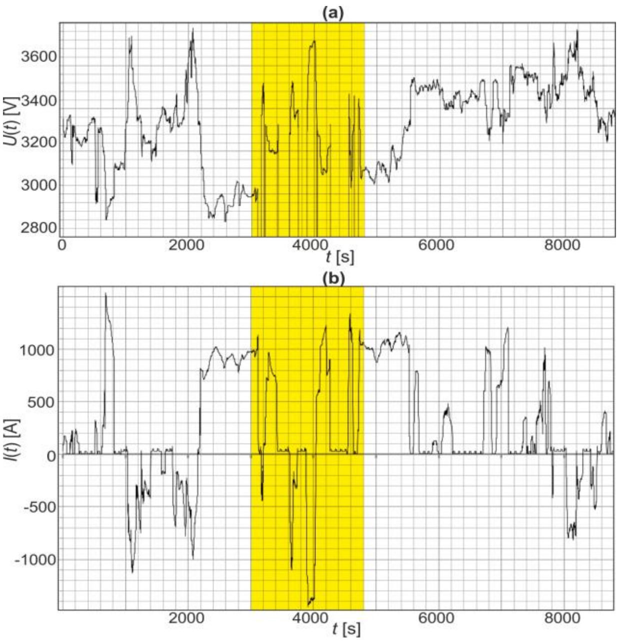

Fig. 1. Time realizations of voltage (a) and current (b) of the DE1 electric locomotive, Ukraine.

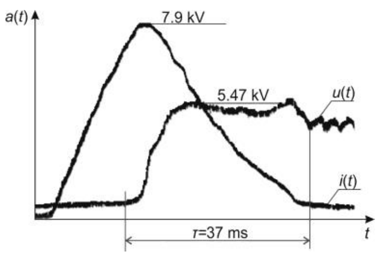

Fig. 2. Voltage and current oscillogram at the interruption moment of short circuit in a traction system by the high-speed circuit breaker of a traction substation, Ukraine.

II. Theoretical Background: Actual and Instantaneous Spectra

In addition to the aforementioned, the main voltage and current fluctuations occur during ERS operation at the moments of load increase and drop, or when the emergency pulses appear and attenuate. This means that it is advisable to estimate the spectrum of a part of the current and voltage pulse, i.e. only their fronts and decays. Therefore, in determining the spectral function in the Fourier integral (1), it is necessary to integrate not within infinite limits, but in some finite interval Δt. For example, if the front spectrum is determined, then such limits should be the time of pulse beginning t_0 and the actual time of front end t (Fig. 3a). It is necessary that the obtained spectral characteristic is not only a function of frequency, but also reflects the investigated process at an actual moment of time, then the spectrum is defined as

$$S_t(j\omega) = \int_0^t f(t)e^{-j\omega t}dt\,, \tag{2}$$

and it can be called as *an actual spectrum* or spectral function of the realization of the stochastic process [17].

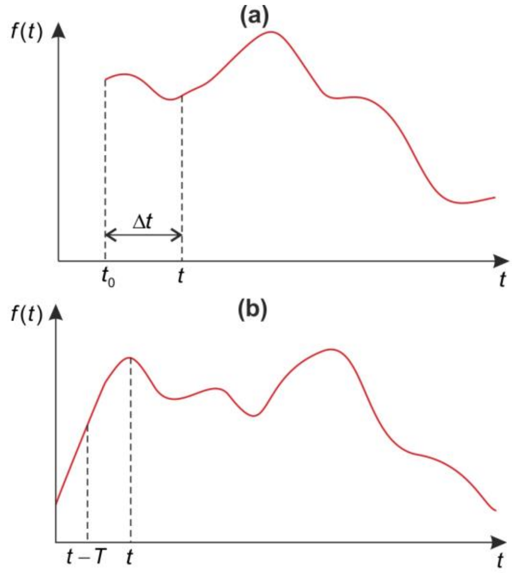

Fig. 3. (a) segment of the time realization of a non-periodic random process for defining its actual spectrum; (b) boundaries of a segment of the time realization of a random process for defining its instantaneous spectrum.

According to (2), the actual spectrum shows all behaviours of the realizations of the random process (up to the time t). That is, the concept of the current spectrum connects the time and frequency descriptions of electromagnetic processes. However, there is a need to deepen and converge the time and frequency view points, and this stipulates a necessity to assume yet another definition – *an instantaneous spectrum* [17] described by the formula

$$S_T(j\omega, t) = \int_{t-T}^{t} f(\tau)e^{-j\omega\tau}d\tau \tag{3}$$

as a spectrum of segment of the process realization of duration T, which is directly preceded by the actual moment of time t (Fig. 3b).

A "sliding" integration is realized in (3) for determination of the instantaneous spectrum, i.e. the interval of integration has a constant length in time, but moves along the time axis. The concept of the instantaneous spectrum not only brings the frequency and time points of view closer together, but also facilitates estimation of the $S(j\omega)$ function for the function's decay, and also, changing in time it shows the behaviours of the process at a given moment.

If we put the "sliding" weight function $\sigma(\tau\text{–}t)$ (associated with the actual time) into the integrand of (3) (Fig. 4), then we obtain a more general definition of the instantaneous spectrum:

$$S_{\sigma}(j\omega,t)=\int_{-\infty}^{\infty}\sigma(\tau-t)f(\tau)e^{-j\omega\tau}d\tau\,. \tag{4}$$

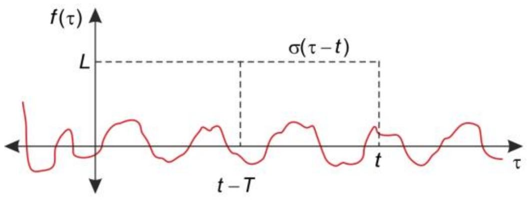

Fig. 4. Time realization of the investigated random process with a weight function $\sigma(\tau\text{–}t)$.

It is easy to see that (3) is a special case of (4), if the weight function is taken in the form of

$$\sigma(x)=r(x+T)-r(x), \tag{5}$$

where $r(x)$ is a unit function.

It follows from (1)-(3) that the instantaneous spectrum can be defined as the difference between two current spectra, i.e. it is defined by the increment that the current spectrum receives over a period of time T:

$$S_T(j\omega,t)=\int_{-\infty}^{t}f(\tau)e^{-j\omega\tau}d\tau-\int_{-\infty}^{t-T}f(\tau)e^{-j\omega\tau}d\tau\,. \tag{6}$$

If T is sufficiently short, the instantaneous spectrum can be estimated as

$$S_T(\omega,t)\approx\frac{\partial S_t}{\partial t}\cdot T\approx\frac{\Delta S_t}{\Delta t}\cdot T\,. \tag{7}$$

A continuation of the development of the instantaneous spectrum theory is the concept of the instantaneous power spectrum $P(\omega,\ t)$ introduced by Chester Page [18] in the form:

$$P(\omega,t)=\frac{\partial}{\partial t}\left|S_t(j\omega)\right|^2. \tag{8}$$

The integral of the instantaneous power spectrum over the entire frequency axis makes it possible to determine the instantaneous power $p(t)$:

$$p(t)=\frac{1}{\Pi}\int_{0}^{\infty}P(\omega,t)d\omega\,. \tag{9}$$

From (8) it follows that the actual spectrum of voltage or current can be determined through the instantaneous power spectrum as:

$$\int_{-\infty}^{t}P(\omega,t)dt=\left|S_t(j\omega)\right|^2. \tag{10}$$

When solving tasks using the concepts of actual and instant spectra, it is necessary to take into account that both types of spectra are continuous, while for analysis of the electromagnetic compatibility of systems a discrete spectrum is necessary. This requires estimation of the frequency range of the continuous spectrum, in which the majority of the pulse energy is concentrated, that is the "effective width of the spectrum" ($\Delta\omega$, Δf). It can be proved that the product of the shortest pulse duration Δt and the smallest width of the spectrum Δf is constant and equal to $\Delta t{\cdot}\Delta f\leqslant 0.46$. This expression functions as a working formula for determining the desired effective width of the spectrum for a given value of Δt.

In the tasks of analysing electromagnetic processes in devices of electric transport systems, it is often necessary to solve the problem of determining the response of a device (or system) from a certain input action; let us consider it with respect to the actual spectrum of a certain value $u(x)$.

The reaction of a linear dynamical system through an pulse transition function $h(x)$ is expressed by a convolution equation

$$u_2(x)=\int_{-\infty}^{x}h(x-\tau)\cdot u_1(\tau)\cdot d\tau\,. \tag{11}$$

Let us assume that the system is stable and for the impact of u_1 there is a Fourier transformation. Then, the current response spectrum of the system is defined as

$$S_{t2}(j\omega)=\int_{-\infty}^{t}u_1(\tau)\cdot d\tau\int_{-\infty}^{x}h(x-\tau)\cdot u_1(\tau)\cdot e^{-j\omega x}\cdot dx\,. \tag{12}$$

Let us analyse the internal integral of (12), replacing in it the upper limit x by ∞. This is legitimate, since, according to the condition of physical feasibility of the system, the value of $h(x\text{–}\tau)$ in the added interval is equal to zero. Then, denoting $x\text{–}\tau=y$, the following is obtained:

$$\int_{-\infty}^{\infty}h(x-\tau)\cdot e^{-j\omega x}\cdot dx=-e^{-j\omega\tau}\int_{-\infty}^{\infty}h(y)\cdot e^{-j\omega y}\cdot dy\,. \tag{13}$$

Taking into account that the direct Fourier transform for the pulse transition function $h(y)$ is the transfer coefficient $H(j\omega)$ of the system, then the double integral given above can be written as

$$S_{t2}(j\omega)=H(j\omega)\int_{-\infty}^{t}u_1(\tau)\cdot e^{-j\omega\tau}\cdot d\tau=H(j\omega)\cdot S_{t1}(j\omega)\,. \tag{14}$$

Thus, the actual spectrum of the output process of the device or the entire linear dynamic system of electric transport is defined as the product of the actual spectrum of the input process and the transmission coefficient of the device or system, respectively.

As far as the traction-regenerative voltages and currents are random processes, each of their realization is a deterministic process, and then short intervals of realization according to Fig. 3 are deterministic too. Taking this into account, let us consider possible methods for determining of the spectra of these segments and pulses, i.e. the methods for estimating their actual and instantaneous spectra.

III. Methods for Estimating of Spectra of Random Pulses

The first method is the method of "discrete electrical engineering". According to this method, the analogue segments of realizations (or pulses) of stochastic processes of traction-regenerative $U(t)$ and $I(t)$ (hereinafter referred to as $f(t)$) have to be discretized by Kotelnikov's theorem [19] at the intervals of time $\Delta t = t_{n+1} - t_n$ in Fig. 5a, where N is the total number of sampling intervals; $n = 1, 2, \ldots, N$; then $\Delta t = T/N$.

As a result of the sampling, we obtain a sequence of δ-pulses multiplied by the value of $f_n = f(n \cdot \Delta t)$ of the function $f(t)$ while taking readings (Fig. 5a):

$$f_\delta(n\Delta t) = \sum_{n=1}^{N} f(n \cdot \Delta t)\delta(t - n\Delta t)\,, \tag{15}$$

or passing on dimensionless sampling intervals, we obtain

$$f_\delta(n) = \sum_{n=1}^{N} f(n)\delta\left(\frac{t}{\Delta t} - n\right). \tag{16}$$

Let us substitute (16) with the formula of spectral density of the n-th rectangular pulse [20], then

$$F(j\omega) = \int_{t_n}^{t_{n+1}} f_\delta(n) e^{-j\omega t} dt \tag{17}$$

or

$$F(j\omega) = \int_{t_n}^{t_{n+1}} \sum_{n=1}^{N} f(n)\delta\left(\frac{t}{\Delta t} - n\right) e^{-j\omega t} dt\,. \tag{18}$$

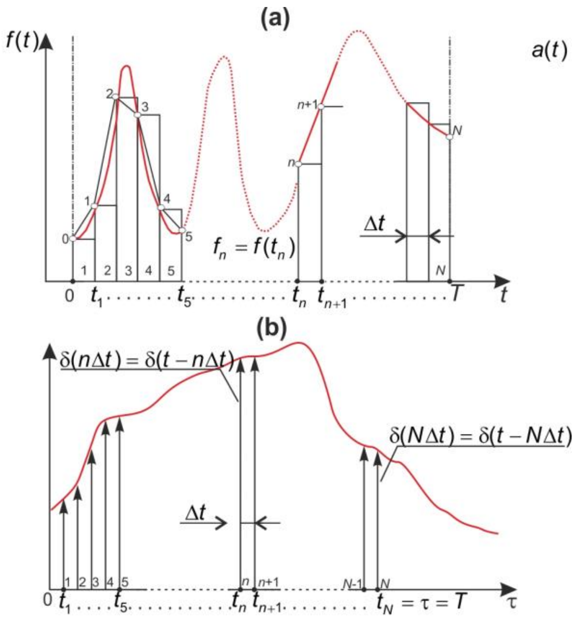

Fig. 5. (a) Approximation of segment of realization of random process $f(t)$ of voltage or current with the sequence of δ-pulses; (b) dispersed pulse function $a(t)$ in the form of a sequence of δ-functions.

Then, let us change the order of integration and addition in (18). We should also consider the filtering property of the delta function, which is that if this function is present in the integral as a multiplier, then the integration result will be equal to the value of another integral function (or expression) at the point (moment of time) where the delta function is concentrated, regardless of the limits of integration. And since the current angular frequency is $\omega = 2\pi / T$, and $t_n = n\Delta t$, then we obtain the desired expression of the spectral density in the form of

$$S(j\omega) = \sum_{n=1}^{N} f(n) \int_{t_n}^{t_{n+1}} \delta\left(\frac{t}{\Delta t} - n\right) e^{-j\frac{2\pi}{T}t} dt = \sum_{n=1}^{N} f(n) e^{-j\frac{2\pi}{T}n\Delta t}\,. \tag{19}$$

From physical point of view, $S(\omega)$ characterises the distribution of the energy of pulse of random process of voltage or current for individual harmonics. It allows determining the frequencies that markedly affect the energy of oscillations in the pulse. Therefore, the function $S(\omega)$ is often called the energy spectrum and is used for estimation of the energy of random electric processes. However, $S(\omega)$ does not contain information about phases of spectral components, nor does it explicitly give the quantitative values of their amplitudes necessary for evaluating the electromagnetic compatibility of systems and the quality of electricity. Therefore, the method given below makes it possible to obtain the necessary ratios.

The second method can be called as a digital spectral analysis and is based on the harmonic analysis of pulses of random processes of voltage or current with application of a discrete Fourier transform. To do this, instead of actually received one pulse $a(t)$ of stochastic voltage or current in Fig. 5b, let us create periodic (with random period T) sequence of pulses. Now, the non-sinusoidal pulse function can be consider not on the interval of its existence $[0 \ldots \tau]$, but on continued periodically outside the interval $[0 \ldots \tau]$. That is, non-periodic pulse function $a(t)$ is transformed into a periodic with period T, for which the expansion in the Fourier series in the real classical form is valid:

$$a(t) = A_{m(k)} \sin\left(k\omega t + \psi_{a(k)}\right), \tag{20}$$

where $A_{m(k)}$, $\psi_{a(k)}$ are the amplitude and the initial phase of the k-th harmonic of the series, which are determined from the complex amplitude $\underline{A}_{m(k)} = A_{m(k)} \mathrm{e}^{-j\psi_{a(k)}}$, based on the known expression in [20]:

$$\underline{A}_{m(k)} = \frac{1}{T}\int_0^T a(t)\mathrm{e}^{-jk\omega t}\, dt\,. \tag{21}$$

However, the function $a(t)$ is a non-sinusoidal, arbitrary and often with very complex form, so application of the classical Fourier analysis is complicated in practice; it is rational to use the discrete Fourier transform. So, as it was done in the first method, let us perform sampling of the pulse function $a(t)$ with the interval of Δt. Then, the values $a_n = a(n \cdot \Delta t)$ are the reference values of already periodic analogue function in the form of a sequence of delta function, "weighed" by reference $a(n \cdot \Delta t)$ of the analogue function $a(t)$:

$$a(t) = \sum_{n=1}^{N} a(n \cdot \Delta t)\delta(t - n \cdot \Delta t)\,. \tag{22}$$

Substituting (22) into (21), we obtain:

$$\underline{A}_{m(k)} = \frac{2}{T}\int_0^T \sum_{n=1}^{N} a(n\cdot\Delta t)\,\delta(t - n\cdot\Delta t)\cdot e^{-jk\omega t}dt\,. \quad (23)$$

As far as values $a(n\cdot\Delta t)$ are constant and independent from t, and function $\delta(t–n\cdot\Delta t)$ is equal to zero for any t except for $t= n\cdot\Delta t$, then (23) can be rewritten as

$$\underline{A}_{m(k)} = \frac{2}{T}\sum_{n=1}^{N} a(n\cdot\Delta t)\int_0^T \sum_{n=1}^{N} \delta(n\cdot\Delta t)\,e^{-jk\omega t}dt\,. \quad (24)$$

Taking into account the filtering effect of the delta function, (16) takes the following form:

$$\underline{A}_{m(k)} = \frac{2}{T}\sum_{n=1}^{N} a(n\cdot\Delta t)\,e^{-jk\omega\Delta t}\,. \quad (25)$$

IV. Results of Numerical Calculations and Their Analysis

Comparative studies of actual and instantaneous spectra, as well as the spectra of realizations and “tails” of the correlation functions were performed for traction voltages and currents of the DE1 and VL8 electric locomotives operated on the sections of Prydniprovsk Railway as well as trams operating on the routes of Dnipro in Ukraine.

The analysis of stochastic changes of voltage on the current collector and the traction current of the aforementioned railway vehicles shows that their basic oscillations occur in the transient modes, i.e. in switching of the traction or coasting mode. This is evidenced by the current spectra of fronts and decays of voltages and currents in Figs. 6a,b and 7a,b in comparison with the spectra of realizations and “tails” of the correlation functions. First, the actual spectra contain harmonics with frequencies 1…9 Hz. At the same time, the frequencies in spectra of realizations of $U(t)$ and in the “tail” of the correlation functions do not exceed 0.11.

Second, the actual spectra of the decays of voltage and current pulses at the same frequencies have a higher level of harmonic amplitudes in 1.5…2.0 times than in their fronts. This is explained by the fact that the derivatives of the traction current *di/dt* and voltage *du/dt* in the switching on traction modes are significantly smaller than in the switching off traction modes.

The instantaneous spectrum of realization of the rheostat tram on time interval from 800 to 860 s is shown in Fig. 8b, and it significantly differs from the spectra of the all realization and the “tail” of the correlation function shown in Fig. 8c,d.

The frequency of the harmonics at the aforementioned interval is 0.25…2.5 Hz, when for entire realization and the correlation function it changes from 10^{-3} to 0.11 Hz. The amplitude of the instantaneous spectrum harmonics is 10 V higher than the amplitudes of spectrum of entire realization (Fig. 8d).

The amplitudes of the harmonics of all kinds of spectra presented in Figs. 6-8 are of a probabilistic nature, that is, they are random quantities. In particular, the statistical distribution of the amplitudes of the voltage spectrum has the probability of 0.19 and according to the Pearson criterion, it gives a log-normal distribution with mathematical expectation $M[\ln U]$=7.82 and standard deviation of 1.96 V.

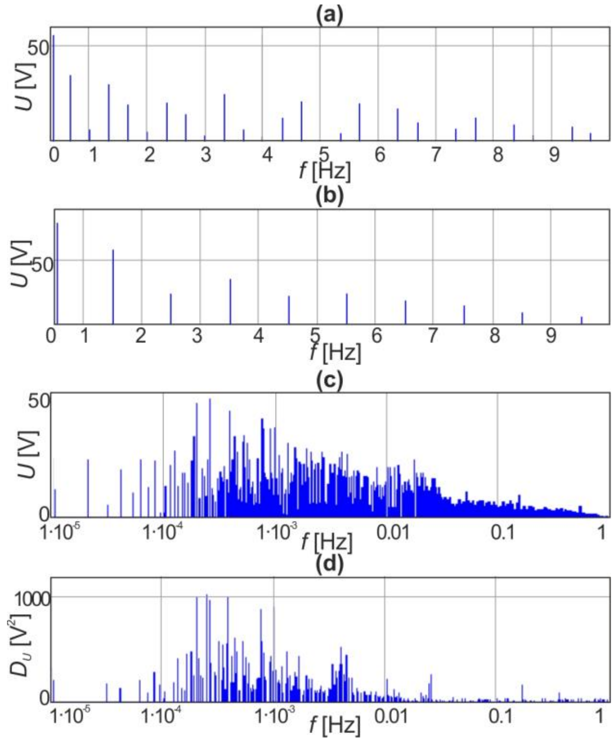

Fig. 6. Actual spectra of voltage on the current collector of the VL8 locomotive in the transients of switching traction modes on (front of voltage pulse) (a) and off (decay of voltage pulse) (b); the spectrum of all the realization of voltage instantaneous curve (c); the spectrum of the “tail” of the voltage correlation function (d).

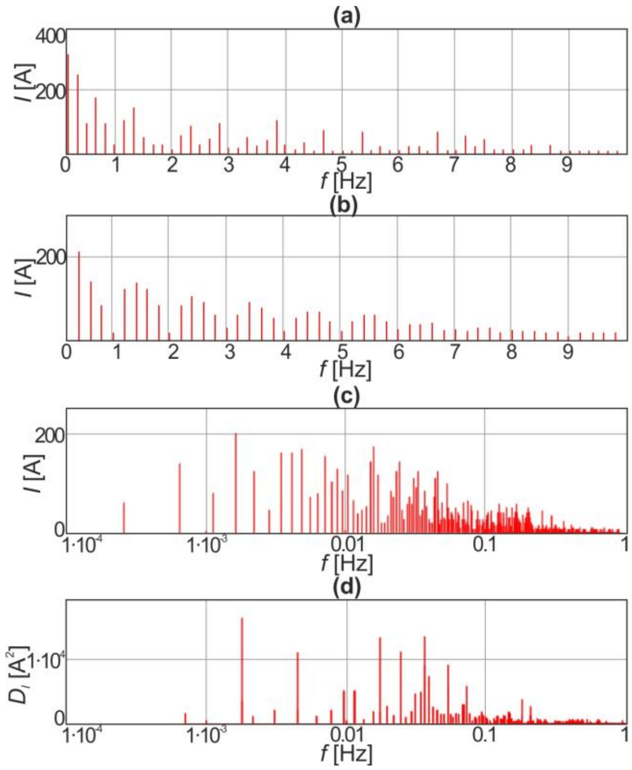

Fig. 7. Actual spectra of current of the VL8 locomotive in the transients of switching traction modes on (front of voltage pulse) (a) and off (decay of voltage pulse) (b); the spectrum of all the realization of current instantaneous curve (c); the spectrum of the “tail” of the current correlation function (d).

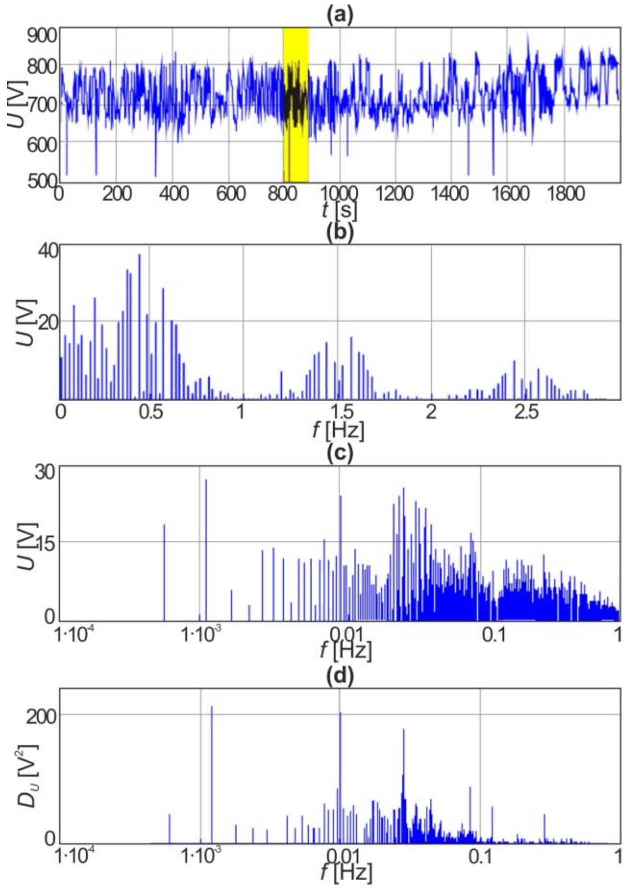

Fig. 8. Voltage realization of a tram with a rheostat control system (a); the instantaneous spectrum of voltage on time interval 800…860 s (b); spectrum of all the realization of voltage instantaneous curve (c); the spectrum of the "tail" of the voltage correlation function (d).

V. CONCLUSIONS

1. The discrete spectra of realizations and "tails" of correlation functions as well as the spectral density of a stationary stochastic process of voltage do not fully characterize the results of its spectral analysis. The use of these values for the analysis of traction-regenerative current is incorrect and even wrong, since the current flow is not a continuous process and really represents a random sequences of short-term non-periodic pulses of arbitrary shape.

2. Introduction of the concepts of actual and instantaneous spectra makes it possible to perform a spectral analysis of the voltage and current not only in a steady-state, but also in non-stationary (transient) and emergency modes of electric traction power supply system operation.

3. The method of discrete Fourier transform is fully applicable to determine the actual and instantaneous spectra.

4. In the actual spectra of the fronts and decays of traction voltages and currents contain harmonics with frequencies exceeding by two orders the frequencies of spectra of all the realizations and "tails" of correlation functions with simultaneous higher amplitudes by 1.5…2 times.

5. The instantaneous spectrum of a 60-second segment of the voltage realization is 0.25…2.5 Hz when the spectra of full realization and the correlation function have frequencies of 0.001…0.11 Hz with simultaneous increase of the harmonics' amplitudes at 10 V.

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