Identification of the railway reconstruction parameters at imposition of high speed traffic on the existing lines

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Abstract. The problems of railroad design technical reconstruction aimed at ensuring train protection in introduction of high-speed traffic on the rail lines have been addressed. The description of railroad technical system with certain specific features has been proposed. The pair of mutually dual problems regarding the optimal reconstruction of railway curves for raising train speeds at minimal capital investments has been addressed. The problem of determining the optimal speeds of trains of each category under optimum outer rail cant, increase of the unbalanced acceleration value, compliance with the condition of uniform load on rail tracks, comfortable carriage of passengers, in which reduced costs on increased speeds in curved tracks will be minimal has been formulated. To achieve the goals of the study, mathematical optimization techniques and algorithms of analysis of the rail lines of different nature are used. This, by finding and value engineering optimization, allows to minimize high-value and lengthy procedures of physical simulation. As a problem-solving technique, Lagrange multiplier method and its applicability have been proposed. Numeral solution to the problem of selection of certain stretches for increasing rolling stock speeds on operating lines without reconstruction or with minimum expenses on reconstruction of contour of track has been reviewed.

1 Introduction

One of the priorities of contemporary railway is raising train speeds on existing rail lines that is inseparably linked to assuring safety and comfortable carriage of passengers.

Economic and social development of the country is directly related to transport development. The economic growth has been accompanied by an increase in the demand for high-speed transport, especially railway one, as it provides citizens of the country with freedom of movement at minimal time expenses. The increase in train speeds produces considerable economic effect through increased line carrying capacity, lower operating costs, labour costs reduction (for locomotive and train crews), reduction of necessary car and locomotive fleet, and costs of fuels and electricity.

One of the major challenges of contemporary railway is reducing travel time of passengers on transregional lines to the maximum permissible, taken in accordance with the condition for ensuring minimal fatigue of organism. For the Republic of Belarus, this criterion shall be 3–5 hours per one way.

Despite the relevance of the problem of the high-speed operation introduction, it is quite difficult to implement it. This can be explained by the lack of the rolling stock, range territory for researches, and regulatory framework, which has just being established.

The works [1-5] are devoted to express and high-speed traffic in foreign countries. The works [6-13] are devoted to the possibility of implementation of high-sped operation on the existing lines.

Features of creation on the railway roads of Ukraine a network of high-speed railway lines are described in the work [6].

The calculation methodology of railway track when interacting with high-speed rolling stock is proposed by D. Kurgan in the work [7].

Theoretical foundations of designing express and high-speed railway lines without reconstruction or with minimal reorganization of existing lines are formulated by A. Gavrilenkov. In the work [9] he noticed that the improvement in design of track contour relates to an increase in the circular curve radius and rail cant.

Analysis of the impact of the unbalanced acceleration on the speed of passenger trains movement is considered

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by P. Kovtun and T. Dubrovskaya. The work [10] proposes one of the options for increasing the speed of passing curves by increasing the permissible limit of the unbalanced acceleration.

The method of determining the technical parameters of the reconstruction of the line for the transition to high-speed passenger trains in mixed freight and passenger traffic, taking into account the uncertainty of the initial information is given in the work [11].

2 Train safety control on railroad transport

The railways must provide not only public demand in passenger transportation, but also passenger safety while travelling on railroad transport.

The specific instruction, which obliges responsible railway workers to pay particular attention to the train safety control, has been developed in order to ensure safe operation of high-speed passenger trains. Special aspects of ensuring safe operation of high-speed passenger trains have been reflected in the various sections, numerous instructions, directions, orders that is poorly adapted for use and awkward while technical training, instructing and trial work period.

Each railway worker connected to handling and service of passenger trains is obligated to fulfil particularly all the operations on preparing entrance, exit and pass routes of passengers trains.

Train safety control in introduction of high-speed operation is the most significant social and economic objective oriented to health care and human security.

Rail traffic safety means an ability of train operation to remain in non-hazardous condition during estimated time, when there is no any life and health threat to passengers, technical stuff, population, safety of cargo, business facilities and transport system technical means.

Increased locomotive capacity and speed used for train traffic lead to increased rate of negative consequences of traffic accidents. For example, the consequences of train or wagon derailment at speeds of 90 km/h and 160 km/h will differ significantly.

Increased magnitude of traffic flow on railroad transport increases the likelihood of negative consequences. Thereof, the more trains pass in unit time, the greater will be the total delay time of trains involuntarily idle due to the emergency stop of one train.

The difficulty of conducting rail traffic safety studies stems from the fact that tens of thousands of people of various specialties, whose activities are connected by unified technological process, take part in the process of transport organization and operation. And traffic safety depends both on those people actions and on technical condition of transport means under their authority.

During research, it is necessary to take into account the linkages and synergies of elements, which may be conditionally combined into four groups:
– human resources (psychophysiological status, professional qualifications, etc.);
– rolling stock (specifications, technical status, etc.);
– rail track (profile, technical status, etc.);
– traffic environment (service position, operation of signalling, communication and electrical supply devices, meteorological conditions, etc.).

Thus, the introduction of high-speed traffic requires comprehensive survey of all the different aspects operating within framework of this system: institutional, technical, psychological, social et. al. The present work reflects an attempt to apply such systematic approach to the problem of train safety control on railroad transport.

3 The relevance of computer optimization methods in introduction of high-speed traffic

Contemporary state of the theory in relation to railroad transport is characterized by the presence of developed system of mathematical models and algorithms for analysis of various characteristics of railroad lines.

An increase in train speeds on existing rail lines using computer engineering means (computer aided design) is one of the major tasks assigned to railroad transport for the foreseeable future.

The use of information technology makes it possible to provide the whole decision-making process at all stages of project development on railroad line reconstruction for high speeds with on-line informational support. Implementing such kind of support requires data source management, the choice of the best features of information model and an effective design management apparatus.

The need for modelling of reconstruction design process itself for express and high-speed railroad lines arises in order to provide a systems analyst with the description tools of design technology being developed.

Application of computer optimization methods in reconstruction of express and high-speed railroad lines will make it possible to search for the optimal solutions in the event of certain statement of the problem without significant material expenses that at this time are one of the most important criteria of each research.

4 The railroad as a simulation object

Being a simulation object, the railroad possesses the following features [9, 13]:
– unpredictability – behavior of subsystems of railroad system is not fully known to researcher. The knowledge of railroad system behavior in the past does not allow reliably predicted its future behavior. Therefore, any system research is carried out at high degree of uncertainty. Herewith, there is high probability of behavior multivalence of a single subsystem under similar conditions and external effects on railroad system;
– uniqueness – none of the existing rail lines has no parallel in respect of the parameters for the system itself and influence of external environmental on it. Hence the need for building of the model for each rail line separately;

– integrity, i.e. the need to consider railroad system as a whole object allowing for decomposition into subsystems during the modelling phase. There is certain difficulty in the multilevel railroad system decomposition connected with different scale of the processes in time ongoing on various hierarchy levels of component communication of the railroad system. Moreover, each component possesses its own "intellect" behavior in the form of complex decision-making and transferring information algorithm in accordance with hierarchy of control actions on components of lower performance level;
– non-interaction, i.e. the presence of autonomous space-time metric and autonomous principle of statefulness within the railroad system;
– modelability – railroad system may be presented by finite set of the models, each of which reflects a certain feature of its essence.

5 Problem statement

Increase in speeds, while insuring safety and maintaining maximum facilities for passengers, might be accessed by using new rolling stock, strengthening track superstructure, and improving a rail line design.

Suppose railroad as a technological system. The speed on the line is limited by capabilities of technical facilities. Speed limitations vary along the length of the line.

It is necessary to split the line into m consistent subsystems – stretches (railway hauls and interstations) with constant speed limit within them.

The stretch is characterized by parameter set of technical facilities (superstructure, including track switches, constructional works) determining speed limitations on the stretch [12].

After reconstruction activities and fixed equipment modernization of varying degrees (replacement of track switches, reinforcement of bridges, etc.) maximum limit speeds on the scratches must be increased up to 120, 140 or 160 km/h respectively.

The improvement in design of track contour relates primarily to an increase in the circular curve radius, rail cant, as well as the change of the unbalanced acceleration value.

Since, in reality, train flows are characterized by a great heterogeneity (high-speed, express, suburban, freight, long, heavy-tonnage, etc.), there is also a need for determination of optimal speeds of trains of each category within every line stretch.

The objective of this study is to find a common approach to solving two problems:
– the definition of optimal values of design radii increasing the speeds of trains on existing lines at minimal cost of reconstruction and maximum travel time reduction,
– the determination of optimal speeds of trains of each category with the introduction of high-speed traffic on existing railway lines with a change of the unbalanced acceleration value from 0,7 m/s^2 to 0,9 m/s^2 and minimal reduced expenditures.

The tasks should be solved with the unconditional safety and comfort of the passengers.

6 Determination of optimal railway curve radii in introduction of high-speed traffic

Travel time reduction ΔT serves as a quantity indicator of speed increase technical efficiency on the stretch, while capital investments K aimed at fixed equipment modernization is a quantitative indicator of economic efficiency.

Curve radii increase leads to higher train speeds and, as a consequence, travel time reduction ΔT. However, the more curve radius, the higher a need in capital investments K for line reconstruction. In reality, capital investments on reconstruction are limited $K \leq K_0$. The restriction can also be imposed on travel time reduction $\Delta T \geq \Delta T_0$.

Let us consider several mutually dual problems regarding the optimal reconstruction of railway curves for raising train speeds at minimal expenditures.

6.1 The problem of optimal radii determination at the maximum travel time reduction

Assume, there is a railway stretch on which m independent (single-radius and compound) curves are located. Each i-curve $(i = \overline{1,m})$ has its:
– curvilinear distance l_i;
– speed limitation within this stretch v_i;
– angle of curvature α_i;
– capital investments K_i for line reconstruction per unit length of curve;
– parameter a dependent on rail cant and acceptable value of unbalanced acceleration.

The aim is to determinate the values of such design radii R_i restricting the speed on curves in which capital investments K will be equal to set K_0, and the travel time reduction ΔT will reach its maximum.

$$\Delta T = \sum_{i=1}^{m} l_i \left(\frac{1}{v_i} - \frac{1}{a\sqrt{R_i}} \right) \to \max \qquad (1)$$

when:

$$\sum_{i=1}^{m} K_i\, \alpha_i R_i^2 = K_0 \qquad (2)$$

To solve the problem of interest, there is a good reason to apply Lagrange multiplier method [14, 15]. First, compose Lagrange function:

$$L(R_i, \lambda) = \sum_{i=1}^{m} l_i \left(\frac{1}{v_i} - \frac{1}{a\sqrt{R_i}} \right) + \lambda \left(K_0 - \sum_{i=1}^{m} K_i\, \alpha_i R_i^2 \right), \quad (3)$$

where λ – Lagrange multiplier representing how much increase in the value of capital investments K_0 per unit will change the maximum reduction of travel time ΔT in optimal resolution while increasing capital investments K_0 per unit.

Let us identify partial derivative of Lagrange function for unknowns R_i $(i=\overline{1,m})$ and λ, and set them to zero. As a result, we obtain the following system of equations:

$$\begin{cases} \dfrac{\partial L(R_i,\lambda)}{\partial R_i} = \dfrac{l_i}{2\,a\,R_i^{3/2}} - 2\,\lambda\, K_i\,\alpha_i R_i = 0, \\ \dfrac{\partial L(R_i,\lambda)}{\partial \lambda} = K_0 - \sum\limits_{i=1}^{m} K_i\,\alpha_i R_i^2 = 0. \end{cases} \tag{4}$$

After solving the system of equations (4) in unknowns R_i and λ for the set value of the capital investments K_0, optimal values of design radii R_i (5) and the maximum reduction of travel time ΔT (6):

$$R_i = \left(\frac{l_i}{K_i\alpha_i}\right)^{2/5} \frac{K_0^{1/2}}{\left(\sum\limits_{i=1}^{m} l_i^{4/5}(K_i\alpha_i)^{1/5}\right)^{2/5}} \tag{5}$$

$$\Delta T = \sum_{i=1}^{m} l_i \left(\frac{1}{v_i} - \frac{1}{a}\left(\frac{K_i\alpha_i}{l_i}\right)^{1/5} \frac{\left(\sum\limits_{i=1}^{m} l_i^{4/5}(K_i\alpha_i)^{1/5}\right)^{1/5}}{K_0^{1/4}} \right) \tag{6}$$

6.2 The problem of optimal radii determination at the maximum capital investments

The following setting of the dual problem of optimal reconstruction of railway curves could be considered:

It is necessary to define values of radii R_i $(i=\overline{1,m})$ restricting the speed on curves in which travel time reduction ΔT will be equal to set ΔT_0, and the capital investments K will be minimal:

$$K = \sum_{i=1}^{m} K_i\,\alpha_i R_i^2 \to \min \tag{7}$$

when:

$$\sum_{i=1}^{m} l_i \left(\frac{1}{v_i} - \frac{1}{a\sqrt{R_i}} \right) = \Delta T_0 \tag{8}$$

Let us compose Lagrange function:

$$L(R_i,\lambda) = \sum_{i=1}^{m} K_i\,\alpha_i R_i^2 + \\ + \lambda\left(\Delta T_0 - \sum_{i=1}^{m} l_i \left(\frac{1}{v_i} - \frac{1}{a\sqrt{R_i}} \right)\right), \tag{9}$$

where λ – Lagrange multiplier, representing how much the change of travel time reduction ΔT_0 per unit will decrease the value of capital investments K in optimal resolution.

As a result, we obtain the following system of equations:

$$\begin{cases} \dfrac{\partial L(R_i,\lambda)}{\partial R_i} = 2\,K_i\,\alpha_i R_i - \lambda \dfrac{l_i}{2\,a\,R_i^{3/2}} = 0, \\ \dfrac{\partial L(R_i,\lambda)}{\partial \lambda} = \Delta T_0 - \sum\limits_{i=1}^{m} l_i \left(\dfrac{1}{v_i} - \dfrac{1}{a\sqrt{R_i}} \right) = 0. \end{cases} \tag{10}$$

After solving the system of equations (10) for the set reduction of travel time ΔT_0, the optimal values of design radii R_i (11) and minimal value of capital investments K (12) are obtained:

$$R_i = \left(\frac{l_i}{K_i\alpha_i}\right)^{2/5} \left[\frac{\sum\limits_{i=1}^{m} l_i^{4/5}(K_i\alpha_i)^{1/5}}{a\left(\sum\limits_{i=1}^{m} \dfrac{l_i}{v_i} - \Delta T_0\right)^{2/5}} \right]^2 \tag{11}$$

$$K = \sum_{i=1}^{m} K_i\,\alpha_i \left(\frac{l_i}{K_i\alpha_i}\right)^{4/5} \left[\frac{\sum\limits_{i=1}^{m} l_i^{4/5}(K_i\alpha_i)^{1/5}}{a\left(\sum\limits_{i=1}^{m} \dfrac{l_i}{v_i} - \Delta T_0\right)^{2/5}} \right]^4 \tag{12}$$

6.3 Numerical solution to the problem of the optimal radii determination

Quantitative solution to the problem of determining the optimal reconstructed radii R_i, using the Lagrangian approach, will be considered on the example of Belarusian Railways section on the Minsk – Krasnoye direction, with a length of 10 km (712 km – 722 km), involving nine independent curves with the radius $R < 2000$ m. Required for solving the set problem characteristics of these curves are presented in Table 1.

Table 1. Characteristics of curves.

№	l, m	ν, m/s	α	K_i, y.e.	a, m/s ^{2}	ΔT, s
1	171,11	40,9	9,73	97777	0,7	3,5
2	226,07	33,3	20,73	129183		
3	436,38	31,3	40,72	249360		
4	196,12	31,4	18,37	112069		
5	245,22	32,1	22,68	140126		
6	174,41	32,3	15,60	99663		
7	183,70	30,8	15,10	104971		
8	522,27	31,8	48,13	298440		
9	305,66	36,8	21,10	174663		

Using computer algebra system MathCAD, the values of optimal radii R_i = (3595; 2656; 2028; 2788; 2563; 2976; 3015; 1897; 2638), are obtained (Figure 1).

$$m := 8 \quad i := 0..m \quad \Delta T := 3.5 \quad a := 0.7$$

$$l := (171.11 \;\; 226.07 \;\; 436.38 \;\; 196.12 \;\; 245.22 \;\; 174.41 \;\; 183.70 \;\; 522.27 \;\; 305.66)^T$$

$$K := (97777 \;\; 129183 \;\; 249360 \;\; 112069 \;\; 140126 \;\; 99663 \;\; 104971 \;\; 298440 \;\; 174663)^T$$

$$\alpha := (9.73 \;\; 20.73 \;\; 40.72 \;\; 18.37 \;\; 22.68 \;\; 15.60 \;\; 15.10 \;\; 48.13 \;\; 21.10)^T$$

$$\upsilon := (40.9 \;\; 33.3 \;\; 31.3 \;\; 31.4 \;\; 32.1 \;\; 32.3 \;\; 30.8 \;\; 31.8 \;\; 36.8)^T$$

$$R_i := \left(\frac{l_i}{K_i \cdot \alpha_i}\right)^{\frac{2}{5}} \cdot \left[\frac{\sum_{k=0}^{m} (l_k)^{\frac{4}{5}} \cdot (K_k \cdot \alpha_k)^{\frac{1}{5}}}{a \cdot \left(\sum_{k=0}^{m} \frac{l_k}{\upsilon_k} - \Delta T\right)}\right]^2$$

$$R^T = \left(3.595 \times 10^3 \;\; 2.656 \times 10^3 \;\; 2.028 \times 10^3 \;\; 2.788 \times 10^3 \;\; 2.563 \times 10^3 \;\; 2.976 \times 10^3 \;\; 3.015 \times 10^3 \;\; 1.897 \times 10^3 \;\; 2.638 \times 10^3\right)$$

Fig. 1. Quantitative solution to the problem using computer algebra system MathCAD

7 The problem of determining the optimal train speeds in curves

7.1 The impact of unbalanced acceleration in introduction of high-speed traffic

The train speed on curved tracks may also be enhanced through increasing of unbalanced acceleration in curves from its acceptable value of 0,7 m/s^2.

In this case, an acceptable value of unbalanced acceleration for rolling stock is determined by:
– human exposure;
– dynamic strength properties of the rolling stock;
– shear-resisting indicators of the track.

Functional state of a passenger is an integral quality indicator of organism properties that define directly or indirectly comfort level of passenger travelling in rolling stock.

Studies have shown that the majority of people can experience continuous and repeated exposure of unbalanced acceleration of up to 0.9 m/s^2 inclusive satisfactorily.

According to studies, experienced by passenger feeling of travel sickness, nausea and dizziness caused by unbalanced acceleration of 0.9 m/s^2 is increased by 0.05 c.u. and considered acceptable.

Unbalanced centrifugal acceleration of 1 m/s^2 at exposure of rare and short-term character can be experienced satisfactorily.

General functional state and working efficiency of locomotive crew at integrated effect of noise, vibration and unbalanced acceleration of 0.9 m/s^2 remains within the tolerances. Thus, increase in unbalanced acceleration to 0,9 m/s^2 will not have significant impact on organism of a passenger, but it can substantially reduce travel time owing to faster running through a curve [10].

The data obtained are enabled to make calculations at unbalanced acceleration of 0.9 m/s^2 and analyse the impact of the unbalanced acceleration increase on speeds of passenger train while rounding a curves of different radii.

Limitation of unbalanced acceleration acceptable rate in terms of dynamic and strength profile is established on the basis of provision on unacceptability of the violation of current requirements to the relevant safety indicators.

The acceptable values of unbalanced acceleration on indicators of impact on the track are determined, taking into account adjustments of individual indicators.

Limitation of train speed and unbalanced acceleration value in curve stretches of track are determined through the occurrence of the limitation in at least one of the foregoing indicators.

7.2 Setting of the problem of determining the optimal train speeds in curves

The problem of determining the optimal speeds of train of each category in curves may be formulated as follows:

Assume, there is a railway stretch with m independent (single-radius or compound) curves on it and running trains of j-category ($j = \overline{1,n}$).

Each curve has its:
– length of curve l (m);
– angle of curvature α;
– curve radius R (m).

It is necessary to determine speeds v_j of the j-category trains ($j = \overline{1,n}$) on a curve of certain radius at optimal rail cant h (mm), increased unbalanced acceleration a, compliance with the condition of uniform load on rail tracks, comfortable carriage of passengers in which reduced costs on increased speeds in curved tracks will be minimal:

$$E = f(v_1, v_2, ..., v_n, h, a) = l\sum_{j=1}^{n} c_j \frac{N_j}{v_j} \rightarrow \min \qquad (13)$$

in restricting:
– compliance with the condition of uniform load on rail tracks:

$$\sum_{j=1}^{n} \beta_j v_j^2 = 12{,}96\frac{ghR}{S}, \qquad (14)$$

– compliance with the condition of comfortable carriage of passengers:

$$h = 12,5\frac{v_{\max}^2}{R} - \frac{S}{g}a, \tag{15}$$

– acceptable values of unbalanced acceleration:

$$a \geq 0,7, \tag{16}$$

$$a \leq 0,9. \tag{17}$$

where c_j – present value of a j-category train;
N_j – the number of trains of j-category;
β_j – weight factor of a j-category train;
g – gravity acceleration, m/s^2;
S – distance between the track axes, mm;
$v_{\max}$ – maximum speed in this curve, km/h;
a – unbalanced acceleration, m/s^2.

7.3 Solution to the problem of determining the optimal train speeds in curves using Lagrange multiplier method

In order to solve the problem of interest (13) – (17), it is appropriate to apply Lagrange multiplier method [14,15].

As the constraints within the problem take the form of equities (14) – (15) and inequalities (16) – (17), to find an absolute minimum of a function f, it is necessary to:

1. Solve the problem when all the constraints, taking the form of inequalities (16) – (17), are omitted, and then define f to each solution.

2. Add one more inequality to the initial constraint system, them another. It is necessary to consider only those solutions that meet all the constraints.

Thus, first we will be considering the problem (13) – (15) excluding the constraints (16) and (17).

Let us compose Lagrange function:

$$\begin{aligned} L(v_1, v_2, ..., v_n, h, \lambda_1, \lambda_2) = l\sum_{j=1}^{n} c_j \frac{N_j}{v_j} + \\ + \lambda_1\left(12,96\frac{ghR}{S} - \sum_{j=1}^{n}\beta_j v_j^2\right) + \\ + \lambda_2\left(12,5\frac{v_{\max}^2}{R} - 163\,a - h\right), \end{aligned} \tag{18}$$

Then identify partial derivative of Lagrange function for unknowns v_j $(j = \overline{1,n})$, h, λ_1, λ_2, and set them to zero. As a result, we obtain the following system of equations:

$$\begin{aligned} \frac{\partial L(v_1, ..., v_n, h, a, \lambda_1, ..., \lambda_4, x_1, x_2)}{\partial v_j} = \\ = -l\,c_j\frac{N_j}{v_j^2} + 2\,\lambda_1\,\beta_j v_j = 0, \end{aligned} \tag{19}$$

$$\begin{aligned} \frac{\partial L(v_1, ..., v_n, h, a, \lambda_1, ..., \lambda_4, x_1, x_2)}{\partial h} = \\ = \lambda_1\,12,96\frac{gR}{S} - \lambda_2 = 0; \end{aligned} \tag{20}$$

$$\begin{aligned} \frac{\partial L(v_1, ..., v_n, h, a, \lambda_1, ..., \lambda_4, x_1, x_2)}{\partial \lambda_1} = \\ = 12,96\frac{ghR}{S} - \sum_{j=1}^{n}\beta_j v_j^2 = 0; \end{aligned} \tag{21}$$

$$\begin{aligned} \frac{\partial L(v_1, ..., v_n, h, a, \lambda_1, ..., \lambda_4, x_1, x_2)}{\partial \lambda_2} = \\ = 12,5\frac{v_{\max}^2}{R} - 163\,a - h = 0. \end{aligned} \tag{22}$$

As follows from the expression (19):

$$l\,c_j\,N_j = 2\,\lambda_1\,\beta_j v_j^3; \tag{23}$$

$$v_j = \left(\frac{l\,c_j\,N_j}{2\,\lambda_1\,\beta_j}\right)^{\frac{1}{3}}. \tag{24}$$

Substitute (24) into (21) and find $\frac{1}{2\,\lambda_1}$:

$$\sum_{j=1}^{n}\left(\frac{l\,c_j\,N_j}{2\,\lambda_1\,\beta_j}\right)^{\frac{2}{3}}\beta_j = 12,96\frac{ghR}{S}; \tag{25}$$

$$\sum_{j=1}^{n}\left(\frac{l\,c_j\,N_j}{2\,\lambda_1}\right)^{\frac{2}{3}}\beta_j^{\frac{1}{3}} = 12,96\frac{ghR}{S}; \tag{26}$$

$$\left(\frac{1}{2\,\lambda_1}\right)^{\frac{2}{3}} = \frac{12,96ghR}{S\sum_{j=1}^{n}\left(l\,c_j\,N_j\right)^{\frac{2}{3}}\beta_j^{\frac{1}{3}}}; \tag{27}$$

$$\frac{1}{2\,\lambda_1} = \left(\frac{12,96ghR}{S\sum_{j=1}^{n}\left(l\,c_j\,N_j\right)^{\frac{2}{3}}\beta_j^{\frac{1}{3}}}\right)^{\frac{3}{2}}. \tag{28}$$

Based on (22) and (28), we obtain:

$$v_j = \left(\frac{l\,c_j\,N_j}{\beta_j}\right)^{\frac{1}{3}}\left(\frac{12,96ghR}{S\sum_{j=1}^{n}\left(l\,c_j\,N_j\right)^{\frac{2}{3}}\beta_j^{\frac{1}{3}}}\right)^{\frac{1}{2}}. \tag{29}$$

Optimal speeds of j-category trains in curves of certain radius at optimal rail cant, acceptable values of unbalanced acceleration, compliance with the condition of uniform load on rail tracks and comfortable carriage of passengers are the following:

$$v_j = \left(\frac{c_j N_j}{\beta_j} \right)^{\frac{1}{3}} \left(\frac{12{,}96 gR\left(12{,}5 \frac{v_{\max}^2}{R} - 163a \right)}{S \sum_{j=1}^{n} \left(c_j N_j \right)^{\frac{2}{3}} \beta_j^{\frac{1}{3}}} \right)^{\frac{1}{2}}. \quad (30)$$

Taking into account the optimal speeds of j-category trains in curve of certain radius, the amount of reduced expenditure will be minimal and of:

$$E = l \sum_{j=1}^{n} c_j N_j \left[\left(\frac{c_j N_j}{\beta_j} \right)^{\frac{1}{3}} \left(\frac{12{,}96 gR\left(12{,}5 \frac{v_{\max}^2}{R} - 163a \right)}{S \sum_{j=1}^{n} \left(c_j N_j \right)^{\frac{2}{3}} \beta_j^{\frac{1}{3}}} \right)^{\frac{1}{2}} \right]^{-1} \quad (31)$$

Varying the values of unbalanced acceleration from 0.7 to 0.9, it is possible to define the optimal speeds of the trains of each category at which reduced costs on increase of the speeds in curves will be minimal.

Conclusions

The problems reviewed above for each line stretch make it possible to determine:
– optimal values of design radii restricting the curve speeds at minimal capital investments and maximum travel time reduction;
– maximum allowed speed of the trains of different category in curves at optimal rail cant, increased unbalanced acceleration from its acceptable value, compliance with the condition of uniform load on rail tracks and comfortable carriage of passengers when reduced expenditures on increase of the speeds in curves will be minimal.

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