# Fryze Reactive Power in Electric Transport Systems with Stochastic Voltages and Currents

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*Abstract*— the paper presents the theoretical foundations of reactive power and additional power losses in a direct current electric transport system. Numerical calculations of inactive power and losses in the circuits of electric DC locomotives and trams have been carried out. Discrepancy between the sign of the instantaneous power and the presence of exchange processes in non-linear circuits of electric transport systems with stochastic voltages and currents is discussed.

## Keywords—instantaneous power, Fryze reactive power, stochastic current, voltage, electric transport, power losses

## I. INTRODUCTION

The problem of energy efficiency which is directly related to evaluation of quality and load balancing in the devices and systems, is an integral part of the analysis of all power components of their electric circuits and, first of all, of reactive power determination. Electric transport systems are not the exception in this problem. Almost half of main-line and suburb transport, all mine and open cast mine railway transport, and city electric transport of Ukraine is exploited using the direct current. Electric locomotives, electric multiple-unit trains, motor-carriages of undergrounds railways, trams and trolley-buses are supplied with direct voltage with rating value from 550 up to 3000 V. In addition, in such type systems there is a lack of reactive power, and the term of power factor can't be applied to them. Therefore, electric consumption is determined only by consumption of active power. So, inaccuracy of such methods is connected with the fact that actually existing continuous changes of coefficients of traction current I(t) and supplying voltage U(t) are not taken into account [1]. Such changes have noncyclic stochastic nature and depend on many factors.

Fig. 1 represents the experimentally obtained time dependencies showing how the values U(t) and I(t) vary.



Fig. 1. Time dependencies of change of random functions of voltage on current collector U(t) and traction current I(t) of Ukrainian electric locomotive.

Data to be analyzed were obtained on first Ukrainian main-line electric locomotive DE1 under its exploitation at one of the areas of Cisdnieper Railways. Taking into account power supply system, any unit of electric rolling stock (ERS) can be considered as a nonlinear load which technologically deforms voltage and current wave shapes. It means that nonlinear electric circuits of ERS, named DC circuits (by the nature of supplying voltage), are actually nonlinear AC circuits and further should be investigated. Thus, the goal of the paper is to show the availability of inactive power component in the systems under consideration and to present evaluation methods of total power components in the systems with stochastic electric values.

## II. THEORETICAL BACKGROUND

It has been known [2,3] that main indicators of existence of reactive power in a circuit are the following: inequality of active P and total power S; availability of the mode of electric pumpback from the load to the generator, variation in time of instantaneous total impedance value z(t) on the input terminals of the load. Let's check whether these indicators exist for DC circuits. For this purpose let us consider the methods and approaches how to measure values of the specified indicators.

It has been known, that for electric circuits with deterministic periodic non-sinusoidal currents and voltages the active power P of the device (system) is defined as the sum-total of active powers of all Fourier series harmonics of the values U(t) and I(t):

$$P = \sum_{s=0}^{n} U^{(s)} \cdot I^{(s)} \cos \varphi^{(s)} , \qquad (1)$$

and total power S using formulas for U and I:

$$S = UI , \qquad (2)$$

$$U = \sqrt{\sum_{s=0}^{n} U^{(s)^{2}}},$$
 (3)

$$I = \sqrt{\sum_{s=0}^{n} I^{(s)^2}} .$$
 (4)

In the expressions (1), (3) and (4)  $U^{(s)} I^{(s)}$  are active values of "s" Fourier series harmonics of voltage U(t) and current I(t). But it is known [2] that neither Fourier series nor Fourier integral transformation aren't applied to stochastic functions defined on an infinitely long interval. Only special kinds of stochastic processes of certain durations can be considered as some exceptions. It is possible to perform their spectral analysis with the help of DFT (discrete Fourier transform) or FFT (fast Fourier transform) presented in [4]. So, let's determine power P taking into account the fact that active power is equal to the mean value over a period of instantaneous power p(t):

$$P = \frac{1}{T} \int_{0}^{T} p(t)dt = \frac{1}{T} \int_{0}^{T} U(t)I(t)dt .$$
 (5)

As U(t) and I(t) are non-periodic functions, let's define *P* as an average arithmetic sum of discrete values of instantaneous power  $p_K$  over the travel time in the given area:

$$P = \frac{\sum_{k=1}^{n} p_k}{n} = \frac{\sum_{k=1}^{n} U_k I_k}{n}.$$
 (6)

Simultaneously, effective values of functions U(t), I(t) (to define *S* according to (2)) are determined as mean square values over the travel time:

$$U = \sqrt{\frac{\sum_{k=1}^{n} U_{k}^{2}}{n}}, \ I = \sqrt{\frac{\sum_{k=1}^{n} I_{k}^{2}}{n}}$$
(7)

Availability of electric pumpback mode from the load to the source (as a secondary indicator of existence of reactive power) for linear circuits of sinusoidal current is classically estimated according to the behavior of instantaneous power p(t): if p(t)>0 the electric pumpback is lacking. However, this assessment method of exchange processes in nonlinear circuits of non-sinusoidal current is incorrect [2]. Moreover, in such type circuits even a classical integral expression of reactive power of the circuit of non-sinusoidal current

$$Q = \sum_{k=1}^{n} U_k I_k \sin \varphi_k \tag{8}$$

does not allow to describe energy processes running between the source and the consumer entirely. But, in this case of stochastic changes U(t) and I(t) the formula (9) can't be applied.

Apparently, the most appropriate methods (or exclusive) are the methods which allow to determine reactive power on the assumption of instantaneous values of stochastic functions of voltage U(t) and current I(t). It means that there is a need to consider instantaneous reactive power q(t) [5-10]. One of the approaches to determine g(t) is based on S.Fryze concept [11]. According to it an electric locomotive as a non-linear passive two terminal circuit is replaced by

two paralleled elements: resistive element with resistance R consuming energy and reactive element which doesn't consumes it or accumulates it first and brings it back to a contact system afterwards. Then, for an unspecified time interval [0,..., T] it can be considered as a duration of one trip. Current of electric locomotive is presented as a sum of active and inactive components:

$$I(t) = I_a(t) + I_p(t) \tag{9}$$

Let us multiply left and right parts of the expression (9) by instantaneous voltage U(t):

$$P = \frac{\sum_{k=1}^{n} p_k \sum_{k} U_k I_k}{n} U_k I_a(\mathfrak{H}) + U(t) I_p(t)$$
(10)

Left part of the expression (10) is a total instantaneous power p(t) of electric locomotive, but the right part is instantaneous active power  $p_a(t)$  and instantaneous reactive power q(t). Then,

$$p(t) = p_a(t) + q(t) \tag{11}$$

Instantaneous active power  $p_a(t)$  can be defined as the power consumed by equivalent resistor R of electric locomotive:

$$p_a(t) = RI_a^2(t) = \frac{U^2(t)}{R}.$$
 (12)

Since active power P of electric locomotive

$$P = \frac{1}{T} \int_{0}^{T} p(t)dt = \frac{1}{T} \int_{0}^{T} U(t)I(t)dt$$

is equal to active power consumed by resistor R

$$P_{a} = \frac{1}{T} \int_{0}^{T} p_{a}(t) dt = \frac{1}{T} \int_{0}^{T} U(t) I_{a}(t) dt =$$
  
=  $\frac{1}{T} \int_{0}^{T} \frac{U^{2}(t)}{R} dt = \frac{1}{R} U^{2}$ 

So,  $P_a = P$ , and

$$P = \frac{U^2}{R}, \quad R = \frac{U^2}{P} \tag{13}$$

where U is the effective voltage value on a current collector of electric locomotive.

Substituting (13) into (12) expression (11) will give us:

$$p(t) = \frac{P}{U^2}U^2(t) + q(t)$$
(14)

Here the instantaneous reactive power q(t) of electric locomotive is defined as [12,13]:

$$q(t) = p(t) - \frac{P}{U^2}U^2(t) = U(t)I(t) - \frac{P}{U^2}U^2(t)$$
 (15)

As q(t) defines a rate of change of electromagnetic energy in the system, it means that when  $q(t)\neq 0$ , between a consumer and a source the exchange processes of electromagnetic energy take place, but when q(t)=0 they are lacking.

It should be noted, that in the circuits with stochastic voltages U(t) and currents I(t) the integral method proposed in [13] is very helpful to determine a reactive power. The expression for the given reactive power  $Q_n$  is defined by a change rate of instantaneous impedance of the circuit z(t):

$$Q_n = -\frac{1}{2\omega_n T} \int_0^T \frac{dz(t)}{dt} I^2(t) dt, \qquad (16)$$

or by an instantaneous total admittance y(t)

$$Q_n = \frac{1}{2\omega_n T} \int_0^T \frac{dy(t)}{dt} U^2(t) dt$$
(17)

where in (16), (17)  $\omega_n$  is constant coefficient with dimension  $s^{-1}$ , called as 'frequency of reduction''. As a result we have:

$$z(t) = \frac{U(t)}{I(t)}, \ y(t) = \frac{I(t)}{U(t)}.$$
 (18)

For non-sinusoidal U(t) and I(t) (in the first approximation we'll assume that for constant in sign stochastic U(t) and I(t)) the formulas (16), (17) after quantizing voltage and current for the intervals (Fig.2) can be written as follows:

$$Q_n = -\frac{1}{2\omega_n T} \sum_{k=1}^n \Delta z_k I_k^2 , \qquad (19)$$

$$Q_n = -\frac{1}{2\omega_n T} \sum_{k=1}^n \Delta y_k U_k^2 \tag{20}$$

where  $\Delta z_k$ ,  $\Delta y_k$  are incremental resistance and conductance in "*k*" quantization point respectively. The third indicator of reactive power is defined by formulas (18).

## III. RESULTS OF NUMERICAL CALCULATIONS AND DISCUSSION

Quantitative estimation of indicators of reactive power existence in the systems of DC electric transport was performed through the example of the first Ukrainian electric locomotives DE1, exploited in 3 areas of CisDnieper Railway. For this purpose with the help of onboard computers of working electric locomotives the voltage values on the current collector U(t) and their traction currents I(t) were recorded. Time interval  $\Delta t = t_k+1 - t_k$  (Fig.2) of discretization of stochastic processes U(t) and I(t), recorded for 20 trips were defined by the Kotelnikov theorem:

$$\Delta t \le \frac{0.5}{f_h}$$

where  $f_h$  is maximal (upper) frequency of spectrum of stochastic process equal to (1/60...1/90) s<sup>-1</sup> for U(t) and I(t).



Fig. 2. Qualitative variations of stochastic voltage U(t) and current I(t) in the systems of electric transport

Duration of voltage and current applied under research was nearly 70 min with discretization interval  $\Delta t = 1$  sec. Figure 3 shows values of total power *S* and active power *P* for each of 20 trips, the values *S* and *P* are determined by the formulas (2) and (6). Based on comparing of these values we get the first indicator ( $P \neq S$ ) of reactive power existence. The values of the last one by Fryze approach (inactive total power component) are defined by the formula

$$Q_F = \sqrt{S^2 - P^2} , \qquad (21)$$

presented by the curve in Fig 3.



Fig. 3. Change of active, Fryze's reactive and total power of the Ukrainian electric locomotive DE1 depending on trips on sections of the Ukrainian CisDnieper railway

However, it is known [2, 3, 12] that the reactive power (including Fryze as the most conceptual) as integral characteristic, can't describe the nonsinusoidal electromagnetic processes exchange.

That is why, as an example, for one experiment the instantaneous impedance z(t) (Fig.4,*c*), instantaneous power p(t) (Fig.4,d) and instantaneous reactive power q(t) (16) (Fig.4,e) were determined according to the dependences U(t) and I(t) (Fig.4, a,b).

According to the sign of instantaneous power (p(t)>0), electric pumpback mode is lacking that shows a lack of exchange process. However, such exchange process must take place as power circuits of electric locomotive have powerful nonlinear reactive elements: inductance of armature windings, windings of main and additional poles of traction engines, inductive shunts. Discovered mismatch of the sign of instantaneous power and availability of exchange processes confirms once again that classic approach to analyze the exchange processes is not applicable for nonlinear circuits with non-sinusoidal currents and voltages. Thus, it would be necessary to accept the idea [12] that for a complete description of exchange electromagnetic processes instantaneous reactive power q(t) (Fig.4,e) is to be considered. As q(t) isn't equal to zero during power consumption period, energy exchange processes take place in a DC electric traction system.



Fig. 4. The character of variation during one trip of electric locomotive DE1: voltage (a), current (b), total instantaneous impedance of electric locomotive (c), instantaneous power (d), instantaneous reactive power (e).

As a result of calculations for the series of trips performed by DC electric locomotives and DC trams it has been stated that reactive power consumption in terms of percentage from total one is 53.8...62.5 % for electric locomotives and 61.8...85.4 % for trams. As a result of this their power coefficient is lower than 1 and equal to 0.65...0.87 for electric locomotives and 0.52...0.723 for trams. The consumption of reactive power  $Q^F$  according to (21) defines nonproductive power losses  $\Delta P$  in active resistance *R* of engines defined as:

$$\Delta P = \frac{Q^F}{U^2} R \tag{22}$$

Nevertheless, the components: in electric locomotives – 26.86...34.28 % from total losses in R equal to 5.7...6.5 % from consumed power by engine, and in trams – 43 % from 5.9...7.1 %. So, variable stochastic nature of change U(t) and I(t) leads to reduction of efficiency factor of electric transport, such as electric locomotives to 0.737 when certified value is equal to 0.891.

### **IV. CONCLUSIONS**

Thus, electric transport systems with stochastic voltages and currents are a nonlinear unsteady load consuming Fryze reactive power. This power reduces the energy coefficients of transport systems and causes additional losses of active energy in their traction chains.

The presence of exchange processes in these circuits should be determined by the sign of instantaneous reactive power, and to improve the energy performance of systems it is necessary to take measures to reduce the inactive component of the Fryze power.

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