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Calculation and analytical method for determining the force param-?ters of a crank-slider mechanism (massage 2. Kinetostatics)

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Розрахунково-аналітичний метод визначення силових параметрів кривошипно-повзуного механізму (повідомлення 2. Кінетостатика)

Метою роботи є розробка алгоритму та програми розрахунку енергосилових параметрів кривошипно-повзунного механізму з урахуванням апробованих на практиці та добре зарекомендованих себе: аналітичного методу розрахунку кінематики повзуна та графо-аналітичного методу розрахунку силових параметрів кривошипно-повзунного механізму. Результати досліджень. Розроблено новий метод розрахунку силових параметрів кривошипно-повзунного механізму. Наукова новизна уявляє новий метод розрахунку силових параметрів кривошипно-повзунного механізму, який відрізняється від існуючих методів графо-аналітичних і аналітичних методів використанням нового розрахунково-аналітичного метода визначення лінійних і кутових прискорень кожної з ланок механізму для вирішенням кінетостатичних рівнянь механізму в цілому. Практична значимість. Створений алгоритм дозволяє виконати попередню оцінку силових параметрів механізмів та агрегатів з використанням кривошипно-повзунних механізмів на стадії технологічного проектування, техніко-економічного обґрунтування нових агрегатів або модернізації існуючого обладнання при зміні технологічних режимів, пов'язаних з освоєнням випуску сучасної конкурентоспроможної продукції, зокрема для виготовлення безшовних труб на станах холодної прокатки труб роликами.

Ключові слова: кривошипно-повзунний механізм, кінетостатика механізму, силові параметри, алгоритм розрахунку.

The purpose of the work is to develop an algorithm and a program for calculating the power parameters of the crank-slider mechanism, taking into account the proven in practice and well-recommended: analytical method of calculating the kinematics of the slider and the graph-analytical method of calculating the power parameters of the crank-slider mechanism. Research results. The new calculation method and an algorithm for determining the force parameters of the crank-slider mechanism. Scientific novelty. The new method for the analysis of force parameters of a crank-slider mechanism, which differs from the existing methods of graph-analytical and analytical methods by the use of a new calculation-analytical method of determining the linear and angular accelerations of each link of the mechanism for solving the kinetostatic equations of the mechanism as a whole. Practical value. The created algorithm makes it possible to perform a preliminary assessment of the force parameters of mechanisms and aggregates using crank-slider mechanisms at the stage of technological design, technical and economic calculation of new aggregates or modernization of existing equipment when changing technological regimes related with the production of modern competitive products, in particular, for the production of seamless pipes on mills with rollers for cold rolling of pipes.

Key words: crank-slider mechanism, kinetostatics of the mechanism, force parameters, algorithm of calculation.

In massage 1, the calculation and analytical method for determining the kinematic parameters of the central crank-slider mechanism (CSM) was developed as a representative of one of the five main types of Assur groups of the 2nd class of the 2nd order [1].

The purpose of this work is to develop an algorithm for calculating the force kinetostatic parameters of the CSM links for the MathCAD application program based on the calculation and analytical method for determining the kinematic parameters of the CSM, taking into account the graphic-analytical method of kinetostatic analysis of mechanisms and machines tested in practice [2-4], which will be the initial data for calculating the strength of parts (links) of the CSM mechanism as part of the operation of high-speed machines and units [1].

Let us consider (Fig. 1) the work of CSM on the example of the operation of a pump [1]

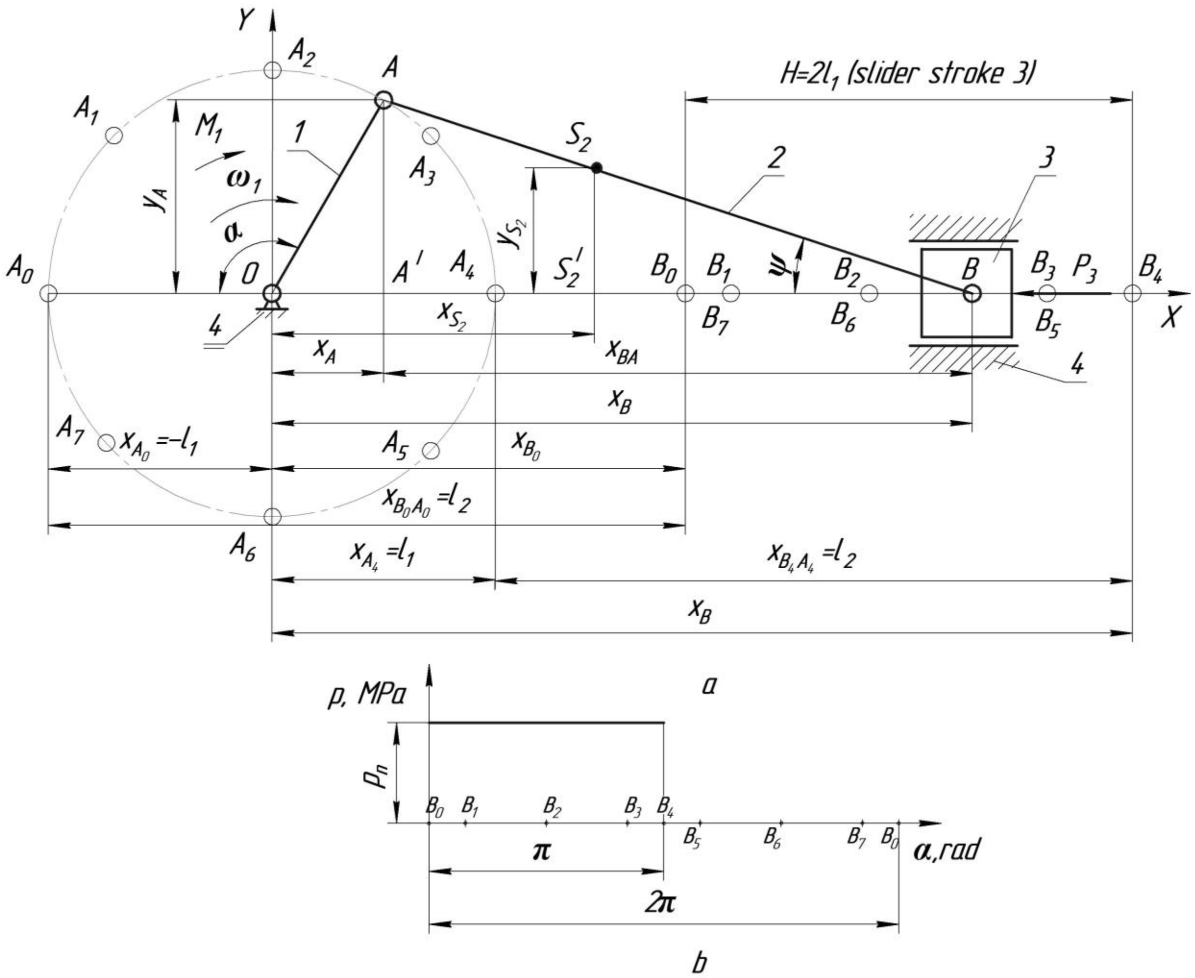


Fig. 1. Scheme of operation of the crank-slider mechanism: *a* – kinematic diagram of CSM; *b* – diagram of slider loading; 3 - during working stroke of CSM

containing a crank (link 1 with length $l_1 = l_{0A}$ is represented in Fig. 1, *a* by the line segment $0A$), which is movable (using hinges 0 and A) and it is connected to a fixed base 4 and connecting rod 2 (link with length $l_2 = l_{AB}$ is represented in Fig. 1, *a* by the line segment AB). Crank 1 rotates uniformly with angular velocity ω_1 (clockwise rotation) relative to the center of the joint O, fixed on a unmovable base 4. The connecting rod 2 performs a plane-parallel motion with the help of a joint B and the rod 2 connected with slider 3, which is located in the fixed horizontal rail of the base 4, and performs a reciprocating motion along the axis $0X$ during crank 1 rotation. In this case, the center of the coordinate system $X0Y$ aligned with the center of the axis of rotation of the joint 0, and the axis $0X$ is directed in the direction of the working stroke of the slider 3 (alone the axis $0Y$), and the slider overcomes external technological resistance P_3, which is created by the pressure ($p_n = \text{const}$) in fluids on slider 3 with the diameter D when it moves in the horizontal guide of the base 4 (Fig. 1, *b*) in the process of turning the crank 1 at the angle $0 \le \alpha \le \pi$ under the action of an external moment M_1 that acts on the crank 1 from the drive side of the CSM, which is on Fig. 1, *a* is not shown. When the crank 1 turns at an angle $\pi < \alpha < 2\pi$ the slider 3 moves towards the axis $0Y$ and idling ($p_n = 0$).

The center of mass m_1 of the crank 1 (with weight $G_1 = m_1 g$) is indicated by a dot S_1. It is located at a distance $l_{OS_1} = 0{,}5 l_{OA}$ from the cylindrical joint O of the crank l_{OA} (line segment OS_1 in Fig. 1, *a*). The center of mass m_2 of the connecting rod 2 (with weight $G_2 = m_2 g$) is indicated by a dot S_2. It is located at a distance $l_{AS_2} = 0{,}35 l_{AB}$ from the cylindrical joint A of the connecting rod l_{AB} (line segment AS_2 in Fig. 1, *a*). Moments of inertia of the crank 1 and connecting rod 2 relative to the centers of mass S_1 and S_2 are $I_{S1} = 0{,}12 m_1 l_1$,

$I_{S2} = 0,12 m_2 l_2$. The center of mass of a slider m_3 (with weight $G_3 = m_3 g$) is a center of a cylindrical joint B. Friction forces in the cylindrical joints 0, A and B as well as between the slider 3 and the base guide 4 are not taken into account in the first approximation.

We take as the reference point ($t_0 = 0$) of the current positions of the links $1, 2, 3$ of the CSM mechanism the position of the initial link 1 when the center of the joint A of the crank 1 is in the position of the working stroke point A_0) from among the intermediate positions of the joint A indicated by the points $A_0 - A_1 - A_2 - A_3 - A_4 - ... - A_7 - A_0$ on the trajectory of the connection rod A along the circle (dash-dotted line in Fig. 1, *a*).

Accordingly, the points $B_0 - B_1 - B_2 - B_3 - B_4 - ... - B_7 - B_0$ determine the current position of the center of the cylindrical joint B of the slider 3 on the axis $0X$ in the horizontal guide of the base 4 (Fig. 1, *a*), which provide the current positions of the connecting rod 2 (line segments $A_0B_0 = A_1B_1 = A_2B_2 = ... = A_6B_6 = A_7B_7 = A_0B_0$ in Fig. 1 are not shown). At the same time points $B_0 - B_1 - B_2 - B_3 - B_4$ determine the current position of the slider during its working stroke, (Fig. 1: *a*, *b*) at which technological force caused by the injected fluid acts on the slider 3 in accordance with diagram of loading (Fig. 1, *b*)

$$P_3 = p_n S_3 = p_n \frac{D^2}{4}, \quad (1)$$

where p_n - the nominal working pressure that the pump creates; S_3 - area of the end surface of the slider 3 with the diameter D.

In its turn, the points $B_4 - B_5 - B_6 - B_7 - B_0$ in Fig. 1: a, b: *a*, *b* determine the current position of the slider 3 when it is idling ($p_n = 0$) in the process of the reciprocating movement of the slider.

The current position of the crank cylindrical joint A of the crank 1 (points A in Fig. 1, *a*) in the process of its uniform rotation ($\omega_1 = \text{const}$) relative to the initial position (points A_0) is determined by the angle

$$\alpha = \omega_1 t_1, \quad (2)$$

where $t_1 > t_0$ the time of rotation of the crank 1 at an angle α relative to the initial position of the crank (line segment OA_0 in Fig. 1, *a*), which is determined by the coordinates of the point A_0 at $t_1 \le T$ - crank rotation period:

$$\begin{cases} x_{A0} = -l_1 \cos\alpha_0 = -l_1 \cos(\omega_1 t_0) = -l_1; \\ y_{A0} = 0. \end{cases} \quad (3)$$

Slider coordinates 3 in this position of CSM (point B_0 in Fig. 1, *a*) will be [1]:

$$\begin{cases} x_{B0} = x_{A0} + l_2 = -l_1 + l_2 = l_2 - l_1; \\ y_{B0} = 0. \end{cases} \quad (4)$$

When crank 1 angle is $\alpha = 0,5\omega_1 T = \pi$ then the point of cylindrical joint A on the trajectory of its movement will take the extreme position along the axis $0X$ and will be located on the circle at the point A_4 with coordinates:

$$\begin{cases} x_{A4} = -l_1 \cos(\pi) = l_1; \\ y_{A4} = 0. \end{cases} \quad (5)$$

Accordingly, the extreme position of the slider 3 at the end of the work stroke determines the coordinate of the point B_4 on the axis $0X$ if $y_{B4} = 0$.

$$x_{B4} = x_{A4} + l_2 = l_1 + l_2.. \quad (6)$$

In this case stroke (H) of the slider 3 will be

$$H = x_{B4} - x_{B0} = l_1 + l_2 - (l_2 - l_1) = 2l_1. \quad (7)$$

In the current position of the crank 1, the coordinates of the point A (the center of the crank joint A) are

$$\begin{cases} x_A = -l_1 \cos\alpha; \\ y_A = l_1 \sin\alpha. \end{cases} \quad 8)$$

From the analysis of right-angled triangles $\Delta OA'A$ and $\Delta OS_1'S_1$ (Fig. 1, *a*) the determination of the coordinates of the current position of the center of mass m_1 (point S_1) of the crank 1 follows:

$$\begin{cases} x_{S1} = x_A \dfrac{l_{OS_1}}{l_{OA}} = 0,5 x_A; \\ y_{S1} = y_A \dfrac{l_{OS_1}}{l_{OA}} = 0,5 y_A. \end{cases} \quad (9)$$

In its turn, the coordinates of the current position of the slider 3 (points B in Fig. 1, *a*) along the fixed horizontal guides 4 (axis $0X$) will be [1-4]:

$$\begin{cases} x_B = OA' + A'B = x_A + x_{BA} = x_A + AB\cos\psi = -l_1 \cos\alpha + l_2 \cos\psi y_B = 0; \\ y_B = 0, \end{cases} \quad (10)$$

where ψ - the current angle that determines the inclination of the connecting rod (the line segment AB in Fig. 1, *a*) to the axis $0X$.

From right triangles $OA'A$ and $BA'A$ it follows $l_1 \sin\alpha = y_A = A'A = l_2 \sin\psi$.

Then

$$\sin\psi = \frac{l_1}{l_2}\sin\alpha = \lambda\sin\alpha. \quad (11)$$

The equation (11) is converted to equation

$$\cos\psi = \sqrt{1-(\sin\psi)^2} = \sqrt{1-(\frac{l_1}{l_2}\sin\alpha)^2} = \sqrt{1-\lambda^2(\sin\alpha)^2} \approx 1-0{,}5\lambda^2(\sin\alpha)^2, \quad (12)$$

where $\lambda = \frac{l_1}{l_2}$ and $(\lambda\sin\alpha)^2 << 1$.

Let's substitute (12) into the first equation of the system of equations (10) and determine the current coordinates of the joint B of the slider 3 on the axis $0X$ at the current position of the initial link 1:

$$\begin{cases} x_B = l_2\sqrt{1-\lambda^2(\sin\alpha)^2} - l_1\cos\alpha \approx l_2(1-0{,}5\lambda^2(\sin\alpha)^2) - l_1\cos\alpha = l_1\left[(\frac{1}{\lambda}-\frac{\lambda}{4})\cos\alpha + \frac{\lambda}{4}\cos 2\alpha\right]; \\ y_B = 0. \end{cases}$$

(13)

From the analysis of right-angled triangles $\Delta AA^{/}B$ and $\Delta S_2S_2^{/}B$ (Fig. 1, *a*) the determination of the coordinates of the current position of the center of mass m_2 (point S_2) of the connecting rod 2 follows

$$\begin{cases} x_{S2} = OA^{/} + A^{/}S_2^{/} = OA^{/} + \frac{A^{/}B}{AB}AS_2 = x_A + \frac{x_{AB}}{l_2}l_{AS_2} = x_A + \frac{x_B - x_A}{l_2}l_{AS_2}; \\ y_{S2} = \frac{OA}{AB}S_2B = y_A\frac{l_2 - l_{AS_2}}{l_1}. \end{cases} \quad (14)$$

The current values of the projections of accelerations ($a_{A,x}, a_{A,y}$) of the crank joint and the projections of acceleration ($a_{B,x}, a_{B,y}$) of the joint B of the slider 3 (Fig. 2) will be determined at

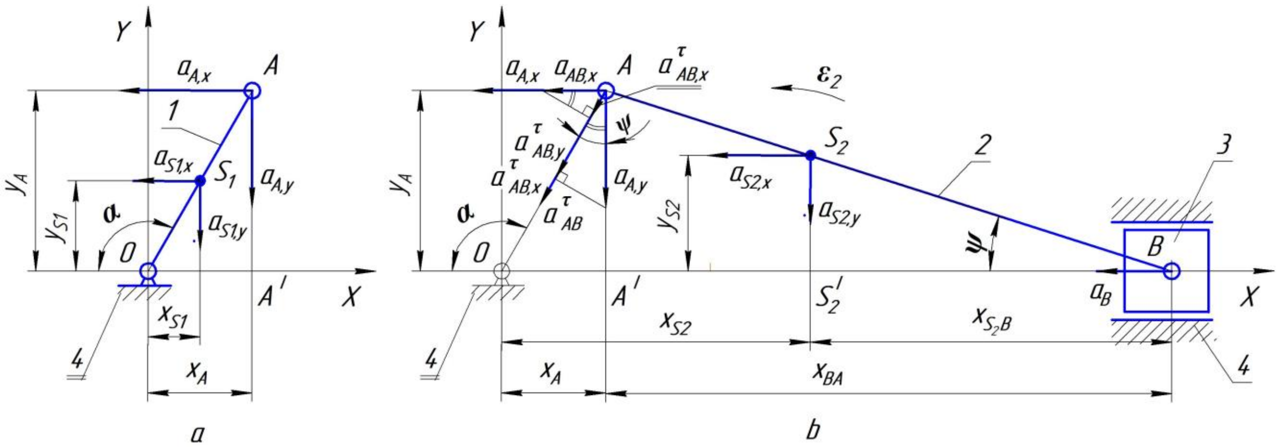


Fig. 2. The distribution of the current values of the projections of the accelerations of the links of the CSM: *a* - the link 1 (crank); *b* – links 2 (connecting rod) and 3 (slider) of the Assur group

successive differentiation of the system of equations (8) and (13).

$$\begin{cases} a_{A,x} = \frac{d^2x_A}{dt^2} = \frac{d^2x_A}{d\alpha^2}\frac{d\alpha^2}{dt^2} = l_1\omega_1^2\sin\alpha, \\ a_{A,y} = \frac{d^2y_A}{dt^2} = \frac{d^2y_A}{d\alpha^2}\frac{d\alpha^2}{dt^2} = l_1\omega_1^2\cos\alpha. \end{cases} \quad (15)$$

$$\begin{cases} a_{B,x} = a_B = \frac{d^2x_B}{dt^2} = \frac{d^2x_B}{d\alpha^2}\frac{d\alpha^2}{dt^2} = -l_1\omega_1^2(\cos\alpha - \lambda\cos 2\alpha); \\ a_{B,y} = 0. \end{cases} \quad (16)$$

The values of the current values of the projections ($a_{S1,x}, a_{S1,y}$) of the acceleration of the center of mass m_1 (points S_1) of the crank 1 are determined (Fig. 2, *a*) from the similarity of right triangles $\Delta OA^{/}A$ and $\Delta OS_1^{/}S_1$. Then:

$$\begin{cases} a_{S1,x} = a_{A,x}\dfrac{l_{OS_1}}{l_{OA}} = 0{,}5a_{A,x}; \\ a_{S1,y} = a_{A,y}\dfrac{l_{OS_1}}{l_{OA}} = 0{,}5a_{A,y}. \end{cases} \quad (17)$$

The values of the current values of the projection ($a_{S2,x}, a_{S2,y}$) of the acceleration of the center of mass m_2 (point S_2) of the connecting rod 2 are determined (Fig. 2, *b*) from the similarity of right triangles $\Delta AA^{/}B$ and $\Delta S_2S_2^{/}B$ [1]. Then:

$$\begin{cases} a_{S2,x} = a_{A,x} + \dfrac{a_B - a_{A,x}}{l_2} l_{AS_2}; \\ a_{S2,y} = a_{A,y}\dfrac{l_{AS_2}}{l_2}. \end{cases} \quad (18)$$

In this case (Fig. 2, *b*), the direction of the relative tangential acceleration (a_{AB}^{τ}) of the joint A relative to the joint B of the connecting rod 2 and, accordingly, the magnitude and direction of the angular acceleration (ε_2) of the connecting rod 2 is determined by the system of equations:

$$\begin{cases} a_{AB}^{\tau} = (a_{A,x} - a_B)\sin(\psi) + a_{A,y}\cos(\psi); \\ \varepsilon_2 = \dfrac{a_{AB}^{\tau}}{l_{AB}}. \end{cases} \quad (19)$$

From (15) - (18) the determination of the current values follows: projections ($P_{i2,x}$, $P_{i2,y}$) of the connecting rod 2 inertia force P_{i2} at the point S_2 and the moment M_{i2} of the connecting rod inertia force (M_{i2}); slider 3 inertia forces P_{i3} at a point B (Fig. 3, *a*); projections ($P_{i1,x}$, $P_{i1,y}$) of the crank inertia force P_{i1} at the point S_1 (Fig. 3, *b*) and the moment of the crank 1 inertia force ($M_{i1} = 0$ at $\omega_1 = \text{const}$):

$$\begin{cases} P_{i2,x} = -m_2 a_{S2,x}; \\ P_{i2,y} = -m_2 a_{S2,y}; \\ M_{i2} = -I_{S2}\varepsilon_2; \\ P_{i3,x} = P_{i3} = -m_3 a_B; \\ P_{i1,x} = -m_1 a_{S1,x}; \\ P_{i1,y} = -m_1 a_{S1,y}. \end{cases} \quad (20)$$

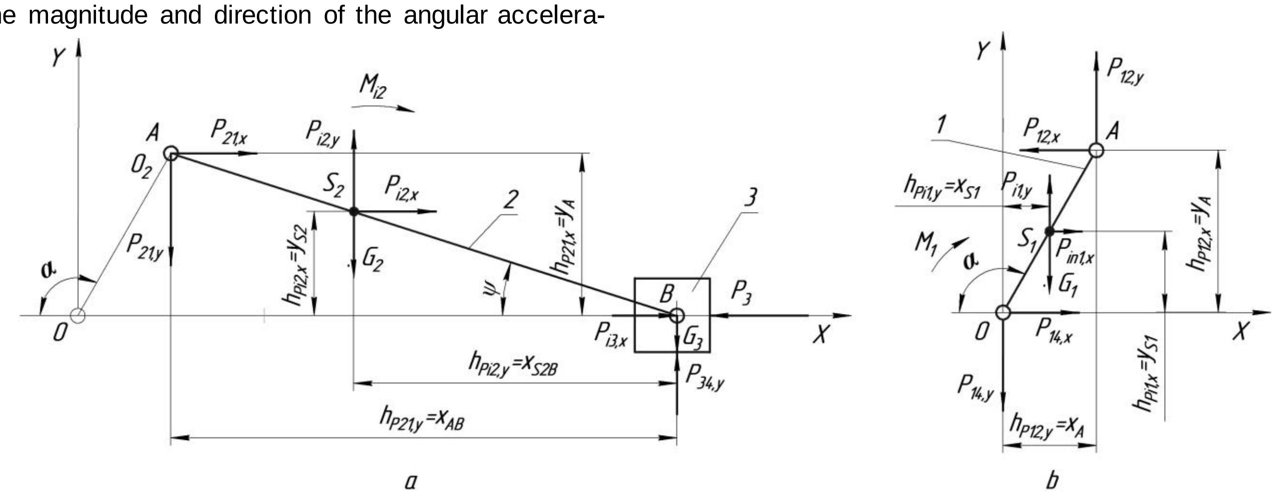


Fig. 3. Calculation scheme of kinetostatic equilibrium during the working stroke of the CSM:
a - Assur group containing links 2 (rod) and 3 (slider); *b* - leading link (crank 1) CSM

When comparing the design scheme (Fig. 3, *a*) of the kinetostatic equilibrium of the Assur group, including links 2, 3 and kinematic connections between the crank 1 and the connecting rod 2 in the joint A, between the connecting rod 2 and the slider 3 in the joint B, as well as between the slider 3 and the horizontal guide of the base 4 in the translational kinematic pair, providing the current position of the links 2 and 3 the CSM in the coordinate system XOY, the kinematic links are conditionally replaced by [2-4] the action of forces ($P_{21,x}, P_{21,y}$) of the force P_{21} on the connecting rod 2 from the side of the crank 1 in the joint A and the action of force ($P_{34} = P_{34,y}$) on the slider 3 from the side of the base guide 4 directed perpendicular to the flat surface of the slider 3 (along the axis OY in the absence of friction sliding between the slider 3 and the guide 4 ($P_{34,x} = 0$)).

Thus, the condition of kinetostatic equilibrium of the links 2 and 3 (Fig. 3, *a*) in the coordinate system is determined by the system XOY of equations:

$$\begin{cases} \Sigma X = P_{21x} + P_{i2,x} - P_3 + P_{i3,x} = 0; \\ \Sigma Y = -P_{21,y} + P_{i2,y} + P_{34,y} - G_2 - G_3 = 0; \\ \Sigma M_B = P_{21,x}h_{P21,x} + P_{i2,x}h_{Pi2,x} + (P_{i2,y} - G_2)h_{Pi2,y} + M_{i2} - P_{21,y}h_{P21,y} = 0, \end{cases} \quad (21).$$

where ΣX and ΣY - respectively, the the sum of the projection of forces on the coordinate axes OX and OY; ΣM_B - the sum of moments of forces and moment of inertia M_{i2} relative to the center of the joint B; $h_{P21,x} = y_A$, $h_{Pi2,x} = y_{S2}$, $h_{Pi2,y} = x_{S2B}$, $h_{P21y} = x_{AB}$ respectively lever arms of forces $P_{21,x}$, $P_{i2,x}$, $P_{i2,y}$ and G_2, $P_{21,y}$ relative to the center of the joint B.

From the system of equations (21), the determination of the kinetostatic loading of the connecting rod 2 and the slider 3 (Assurian group 2, 3) follows, in the coordinate system XOY:

$$\begin{cases} P_{21x} = P_3 - P_{i2,x} - P_{i3,x}; \\ P_{21,y} = \dfrac{P_{21,x}h_{P21,x} + P_{i2,x}h_{Pi2,x} + (P_{21,y} - G_2)h_{Pi2,y} + M_{i2}}{h_{P21,y}}; \\ P_{34,y} = P_{21,y} - P_{i2,y} + G_2 + G_3. \end{cases} \quad (22)$$

In its turn, on the basis of Newton's third law, the components $P_{12,x} = -P_{21,x}$ and $P_{12,y} = -P_{21,y}$ acting in the current position of the working stroke of the CSM on the crank 1 from the side of the connecting rod 2 in the joint A (Fig. 3, *b*), which, together with the components $P_{14,x}$ and $P_{14,y}$ acting on the crank from the side of the base in the joint O; components $P_{i1,x}, P_{i1,y}$ and gravity G_1 provide conditions for uniform rotation of the crank 1 with angular velocity ω_1 due to external moment M_1 during the working stroke ($M_1(\alpha) = M_{ws}(\alpha)$ at $\pi \le \alpha \le \pi$).

The equilibrium condition for link *1* (Fig. 3, *b*) in the coordinate system XOY is determined by the system of equations:

$$\begin{cases} \Sigma X = -P_{12,x} + P_{14,x} + P_{i1,x} = 0; \\ \Sigma Y = P_{12,y} - P_{14,y} - G_1 + P_{u1,y} = 0; \\ \Sigma M_0 = M_1 - P_{12,x}h_{P12,x} - P_{12,y}h_{P12,y} + P_{i1,x}h_{Pi1,x} - (P_{i1,y} - G_1)h_{Pi1,y} = 0, \end{cases} \quad (23)$$

where ΣM_O - the torques sum of the $P_{12,x}$, $P_{12,y}$, $P_{i1,x}$, $P_{i1,y}$, G_1 and moment M_1 relative to the center of the joint O; $h_{P12,x} = y_A$, $h_{P12,y} = x_A$, $h_{Pi1,x} = 0{,}5y_A$ $h_{Pi1,y} = 0{,}5x_A$ - respectively lever arms of forces $P_{12,x}$, $P_{12,y}$, $P_{i1,x}$, $P_{i1,y}$ and G_1, relative to the center of the joint O.

From (26):

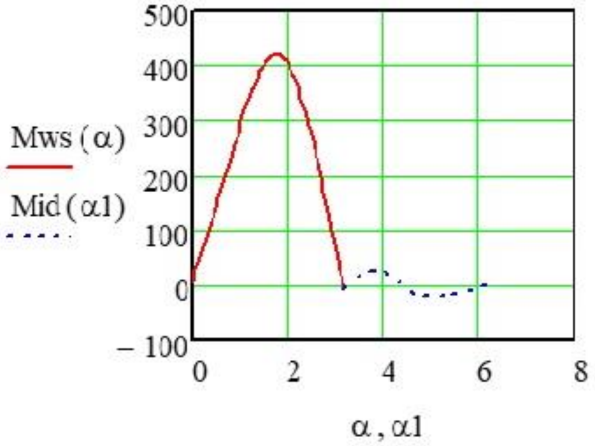
$$\begin{cases} P_{14,x} = P_{12,x} - P_{i1,x}; \\ P_{14,y} = P_{12,y} + P_{i1,y} - G_1; \\ M_1 = P_{12,x}h_{P12,x} + P_{12,y}h_{P12,y} - P_{i1,x}h_{Pi1,x} + (P_{i1,y} - G_1)h_{Pi1,y}. \end{cases} \quad (27)$$

In the kinetostatic calculation of the CSM in the process of idling ($M_1(\alpha) = M_{id}(\alpha_1)$) (during turning the crank at an angle $\pi \le \alpha = \alpha_1 \le 2\pi$), it is necessary to accept in equation (1) $p_n = 0$ and, accordingly, in the equations of the systems of equations (21), (22) accept $P_3 = 0$ and all other design equations (3) - (20) and (23), (24) remain unchanged.

Below (Fig. 4) are the results of change in kinetostatic moment $M_1(\alpha)$, for one revolution of the crank, calculated in accordance with the developed method using the MathCAD application for the following initial data:

$l_1 = 0{,}1$ m; $l_2 = 0{,}285$ m; $l_{AS_1} = 0{,}05$ m; $l_{AS_2} = 0{,}1$ m; $m_1 = 2{,}8$ kg; $m_2 = 3{,}6$ kg; $I_1 = 0{,}342 \cdot 10^{-2}$ kg·m²; $I_2 = 0{,}7 \cdot 10^{-1}$ kg·m²;

$\omega_1 = 29{,}3$ s^{-1}; $p = 29{,}3$ s^{-1}; $p = 0{,}35$ MPa; $D = 0{,}12$ m.



$M_{ws}(\alpha)$ [Nm] - moment on the crank during the working stroke ($0 \le \alpha \le \pi$),

$M_{id}(\alpha_1)$ [Nm] - moment on the crank during idling ($\pi \le \alpha_1 \le 2\pi$).

Fig. 4. Graph of the change in the kinetostatic moment of the leading link (crank 1) CSM:

Conclusions. The new calculation method and algorithm for determining the kinetostatic parameters of the CSM for personal computers using standard MathCAD applications was developed, which allows you to expand the possibilities of studying the reliable operation of the CSM as a part of various machines and units by clarifying the power parameters of the CSM depending on the technological load, dimensions and configuration of parts. and connections.

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