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Mathematical Modeling of the Second Stage of Spring Suspension of High-speed Rolling Stock

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Abstract. The authors of the article used a thermodynamic model of an air spring with an additional reservoir to simulate the second stage of spring suspension of high-speed rolling stock. In this model, the pressure in the air spring was determined by the differential form of the ideal gas equation of state. Since the air spring is connected to an additional reservoir by means of a pipeline, the equation took into account the change in air mass due to its overflow. When determining the mass flow rate, the flow was considered turbulent, which provided for finding the appropriate Reynolds number and head losses, which consisted of three components: friction, compression and expansion losses. The law of conservation of energy was considered taking into account the heat transfer between the air spring and the environment, as well as the transfer of energy between the air spring and the auxiliary reservoir. Using a simplified mechanical model of high-speed rolling stock, the dependence of the force with which the air springs acts on the elements of the rolling stock on its deformation was obtained, and energy dissipation when changing the parameters of the connecting element was investigated. Using of the selected thermodynamic model of the air spring in the spatial model of high-speed rolling stock will further allow assessing its dynamic performance when interacting with the railway track, traffic safety indicators and choosing the optimal parameters of the rolling stock and railway track at the design stage.

INTRODUCTION

The design process is the initial component of modern production of high-speed rolling stock and takes place with the help of computer and mathematical modeling methods [1]. To ensure safe traffic conditions of high-speed electric trains EKr1 «Tarpan» and HRCS2 (Hyundai Rotem), which are operated on the railways of Ukraine, in their second stage of spring suspension, air springs are used, the characteristics of which are substantially nonlinear and significantly affect the frequency spectrum of rolling stock oscillations (Fig. 1).

In the spatial mathematical models used in the design of the relevant units of rolling stock, the features of air springs, as a rule, are not taken into account [2-4]. Air springs are represented by simplified viscoelastic models, and dynamic and traffic safety indicators are determined during operational tests. A more detailed depiction of characteristics of an air spring can be performed using thermodynamic and finite-element (FEM) models, which make it possible to take into account changes in temperature, heat transfer, air flow between the air spring and the additional reservoir, etc.

Thus, the mathematical modeling of a high-speed electric train in its interaction with the rail track, which is based on a more detailed mathematical model of an air spring at the design stage, is a relevant task and needs further solving.

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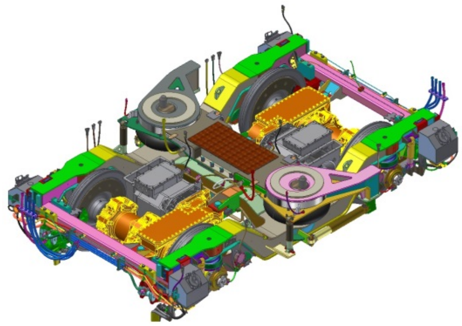


FIGURE 1. Trolley of the high-speed electric train EKr1 «Tarpan»

ANALYSIS OF LITERARY REFERENCES AND CHOICE OF AN AIR SPRING MODEL

For the optimal choice of the mathematical model of an air spring, the analysis of scientific and technical literature was carried out. Thermodynamic and finite-element models were considered, which make it possible to perform the calculation without involving previously obtained experimental data as input parameters of the model.

Thermodynamic models can be divided into different categories depending on the number of described elements (air spring, connecting pipeline, orifice, additional reservoir) and physical phenomena (friction losses, air inertia, the possibility of heat transfer, etc.).

In work [5], a thermodynamic model of an air spring was created, which includes a pneumatic cylinder, a pipeline and a reservoir (Fig. 2). Such model is mainly aimed at the study of vertical oscillations, but does not take into account heat exchange with the environment.

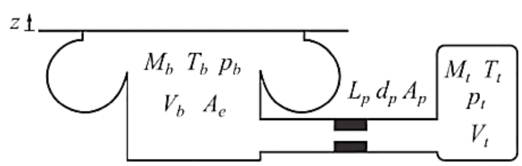
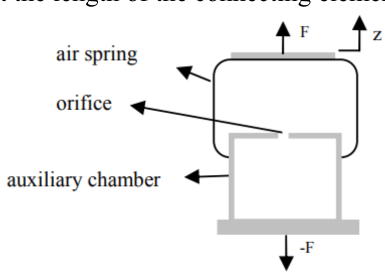


FIGURE 2. Model of an air spring [5]

In contrast to [5], the model presented in work [6] takes into account the phenomenon of heat exchange with the environment. As noted in paper [7], at low oscillation frequencies of an air spring $f < 0.1$ Hz there is an isothermal process, and at frequencies $f > 3$ Hz - an adiabatic process. Since the oscillations of the second stage of the spring suspension of a passenger car occur with a predominant frequency from 0 to 4 Hz, the consideration of heat exchange is important for the main frequency range of work of an air spring [8].

Mathematical models of a pneumatic cylinder and an additional reservoir were given in works [9-10] under the assumption that they are directly interconnected (Fig. 3). Such mathematical model does not allow taking into account the length of the connecting element of an air spring of high-speed rolling stock.



Mathematical model

$$P_s dV_s + V_s dP_s = R\left(m_s dT_s + T_s dm_s\right)$$

$$dQ = dE + P_s dV_s + h dm_s$$

$$dE = C_v m_s dT_s + C_v T_s dm_s$$

FIGURE 3. Pneumatic system without a connecting pipeline

where P_s – absolute pressure in the air spring, Pa; V_s – volume of the air spring, m^3; R – gas constant, $R = 286.9\ \text{J}/(\text{kg}\cdot\text{K})$; m_s – air mass in the air spring, kg; T_s – air temperature in the air spring, K; dQ – heat exchange between the air spring and the environment, $dQ = 0$; dE – internal energy change; h – enthalpy of air, J/kg; C_v – specific heat at constant volume, $\text{J}/(\text{kg}\cdot\text{K})$.

In work [11], the author developed a general mathematical model of an air spring, taking into account heat exchange with the environment (Fig. 4).

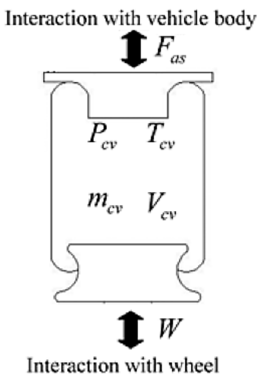


FIGURE 4. Basic parameters of an air spring

Mathematical model of an air spring:

$$\dot{P}_{cv} = -kP_{cv}\frac{\dot{V}_{cv}}{V_{cv}} + \frac{k-1}{V_{cv}}h_c A\left(T_{env} - \frac{V_{cv}}{Rm_{cv}}P_{cv}\right) + \frac{kR}{V_{cv}}\left(T_{in}\dot{m}_{in} - \frac{P_{cv}V_{cv}}{m_{cv}R}\dot{m}_{out}\right)$$

where k – coefficient of specific heat; P_{cv} – pressure in the air spring; V_{cv} – volume of the air spring; h_c – heat transfer coefficient; A – heat transfer area; T_{env} – ambient temperature; R – ideal gas constant; m_{cv} – air mass in the air spring; T_{in} – temperature of air entering the air spring; $\dot{m}_{in}$ – mass flow of air entering the air spring.

However, such model does not take into account the presence of an additional reservoir and a connecting element in the design of a spring suspension of high-speed rolling stock.

One of the most modern methods of analyzing the dynamic behavior of systems and their components is the finite element method [12-15]. The most common software product for modeling air springs is the "Abaqus" package. The finite-element model of a spring is mainly created with the help of ANSA software [16]. The results of the corresponding calculation of an air spring are shown in (Fig. 5).

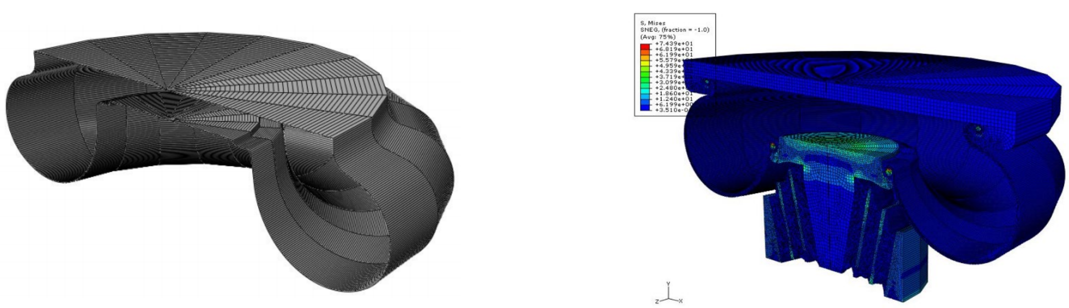


FIGURE 5. Modeling of an air spring in the «Abaqus» software package

To obtain the required accuracy of the finite-element calculation, it is necessary to divide the air springs into a large number of finite elements of different types with different characteristics, which makes the calculation cumbersome and complicates the mathematical model.

In consequence of the conducted analysis, to model an air spring as a part of high-speed rolling stock, the model presented in work [6] was chosen, as the one which reflects the main properties inherent in the processes in spring suspension, does not require experiments to obtain input data and does not overload the overall model.

THE PURPOSE AND TASKS OF THE STUDY

The purpose of this work is to select an adequate model of an air spring on the basis of the analysis of literary references and to create a simplified model of high-speed rolling stock, taking into account the peculiarities of air spring suspension work in the conditions of cyclic loading.

Tasks of the study:

- to analyze the existing mathematical models of an air spring, to note their advantages and disadvantages in the process of modeling high-speed rolling stock and to choose the optimal one for studying the vertical dynamics of railway rolling stock;
- to build a simplified mathematical model of high-speed rolling stock, taking into account the air spring;
- to investigate the influence of air spring parameters on the strength characteristic of the spring suspension in order to prove the adequacy of the chosen mathematical model.

THE CHOSEN MATHEMATICAL MODEL OF AN AIR SPRING

As a result of literary references analysis, a thermodynamic model presented in work [6] was chosen for modeling the dynamics of high-speed rolling stock.

An air spring together with an additional reservoir in the working position are filled with compressed air, which provides its necessary rigidity and damping. Since the relative pressure and temperature are within the limits where we can assume the ideal behavior of the gas, to find the internal pressure of the air spring, the ideal gas equation is used:

$$PV = mRT \tag{1}$$

where P, V, T – pressure, volume and temperature of the working medium of the air spring, respectively; m – mass of air; R – gas constant.

Differentiating equation (1) by time, we obtain:

$$\frac{dP(t)}{dt}V(t) + \frac{dV(t)}{dt}P(t) = \frac{dm(t)}{dt}RT(t) + \frac{dT}{dt}m(t)R \tag{2}$$

The continuity equation, which is included in the differential form of the ideal gas equation, can be expressed:

$$\frac{dm_1(t)}{dt} = -\dot{m}(t) \qquad \frac{dm_2(t)}{dt} = \dot{m}(t) \tag{3}$$

Indices «1» and «2» correspond to the air spring and the additional reservoir, respectively.

In determining the mass flow, a turbulent flow was considered, taking into account the pressure losses due to friction, using the Darcy-Weisbach equation, and the local losses due to instantaneous expansion and contraction.

The Darcy-Weisbach equation has the form:

$$\Delta h = \xi \cdot \frac{V^2}{2g} \tag{4}$$

where ξ – coefficient of friction losses; V – average speed of the working medium movement; g – free fall acceleration.

Knowing the average speed of the working medium movement and carrying out analytical transformations, the mass flow is found by the formula:

$$\dot{m}(t) = \rho(t) A_{per} \sqrt{\frac{2\Delta P(t)}{\rho(t)\left(f\frac{l}{d} + K_s + K_p\right)}} \tag{5}$$

where ρ – density of the working medium; A_{per} – cross-sectional area of the connecting pipeline; l, d – length and diameter of the connecting pipeline; K_s – compression loss coefficient; K_p – expansion loss coefficient.

To find the temperature of the working medium, we use the law of conservation of energy, taking into account the heat transfer between the air spring and the environment, as well as energy transfer between the air spring and the additional reservoir:

$$U(t) = Q(t) - W(t) + E(t) \tag{6}$$

$$\Delta U = mC_v \Delta T(t) \tag{7}$$

$$\Delta Q(t) = h_T A_s(t)(T_s - T(t))\Delta t \quad (8)$$

$$\Delta W(t) = F(t)\Delta h(t) = A_p P(t)\Delta h(t) \quad (9)$$

where U – internal energy of the air spring; Q – heat transfer; W – work done; E – energy transfer between the air spring and the additional reservoir; Δh – deviation of the air spring from the state of equilibrium; A_p – cross-sectional area of the air spring; A_s – heat transfer area; h_T – heat transfer coefficient; C_v – specific heat at constant volume.

Similar formulas are stipulated for the additional reservoir, taking into account the lack of heat transfer to the environment and changes in volume.

RESEARCH OF INFLUENCE OF AIR SPRING PARAMETERS ON STRENGTH CHARACTERISTIC OF SPRING SUSPENSION

A simplified mechanical model with two degrees of freedom was used to model the vertical oscillations of high-speed rolling stock (Fig. 6). Numerical calculations were performed, using «Mathcad» software with the following model parameters:

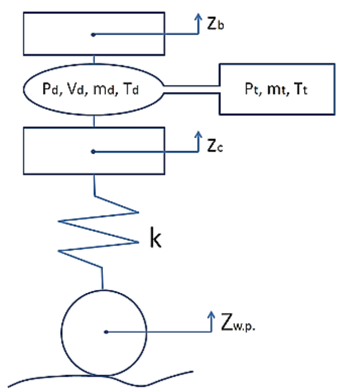


FIGURE 6. Modeling of an air spring in the «Abaqus» software package

- speed of movement – 20 m/s;
- amplitude of vertical irregularity of the rail track – 0.020 m;
- length of irregularity of the rail track – 20 m;
- initial pressure of the air spring – 0.7 MPa;
- diameter of the connecting element was in the range from 5 mm to 20 mm;
- length of the connecting element was in the range from 0.5 m to 1.5 m.

To prove the adequacy of the accepted model of an air spring, its strength characteristic (dependence of force on spring deformation) and the influence on it of changes in the parameters of the connecting element were studied (Fig. 7-8).

(Fig. 7) shows strength characteristics of an air spring with different diameters of the element connecting the air spring with an additional reservoir. Studies show that the optimal range for changing the diameter of the connecting element is from 15 to 20 mm [17].

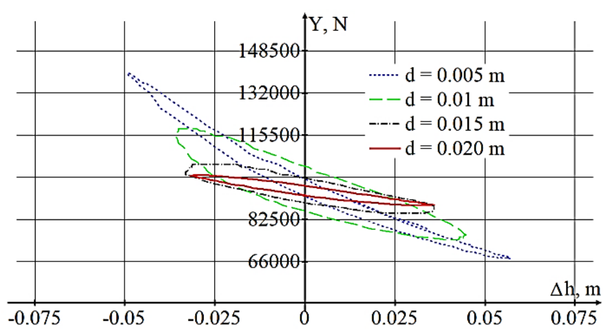


FIGURE 7. Dependence of force of the air spring on its deformation at different values of diameter of the connecting element

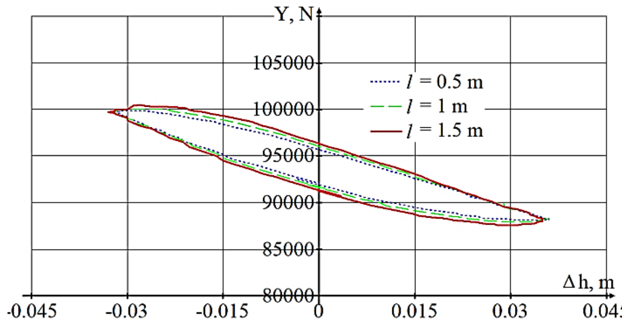


FIGURE 8. Dependence of force of the air spring on its deformation at different values of length of the connecting element

The qualitative picture of dynamic behavior of an air spring, obtained in (Fig. 7-8), corresponds to reality [18], indicating the nonlinearity of relationship between force and deformation of the spring, as well as the significant energy dissipation during its operation. The value of energy dissipation is characterized by the area of the strength characteristic loop. Based on this, it was found that increasing the diameter of the connecting element at a fixed perturbation force frequency (1 Hz) leads to an initial increase in energy dissipation per cycle, and then (in case of a diameter greater than 0.01 m) the dissipation decreases. This can be explained by the reduction of pressure losses in the connecting element. Changing the frequency of the perturbing force does not change the behavior of energy dissipation value, but the maximum dissipation will be observed in case of other diameters of the connecting element.

The obtained strength characteristics make it possible to estimate the average rigidity of the spring as tangent of the angle of inclination of the secant of characteristic, which connects the points of minimum and maximum deformation. Increasing the diameter of the connecting element reduces the dynamic rigidity and brings it closer to static one.

The dependences presented in (Fig. 8) show the influence of the length of the connecting element on the value of energy dissipation in the air spring. As expected, increasing the length of the connecting element leads to an increase in the value of dissipation.

Thus, based on the solution to a dynamic problem for a simplified mathematical model of an electric train, a conclusion is made about the qualitative adequacy of the air spring model, which can be further used in building a spatial mathematical model of high-speed rolling stock.

CONCLUSIONS

The importance of conducting research of high-speed rolling stock taking into account the peculiarities of behavior of an air spring of the second degree of spring suspension at the design stage is emphasized. Construction of the corresponding mathematical model will allow establishing dynamic indicators and indicators of traffic safety without carrying out additional operational tests.

Based on the performed analysis of literary references, the optimal thermodynamic model of an air spring was chosen, built using the equations of ideal gas, continuity and energy balance. Using this model, a simplified model of a high-speed electric train was created.

To check the adequacy of the constructed model and take into account the peculiarities of the air spring work, the dependences of the spring force on its deformation at different values of the diameter and length of the connecting element were obtained. It is established that as the diameter of the connecting element increases, the rigidity of the air spring decreases, approaching the static one. Increasing the length of the connecting element increases the value of energy dissipation. The strength characteristic of the spring has the form of a hysteresis loop, which corresponds to the expected features of the air spring work.

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