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Estimation of strength properties of functionally graded structures with elliptical stress concentrators

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ABSTRACT

Purpose: The purpose of this paper is to elaborate new calculation schemes for evaluating the strength parameters of railway rolling stock parts with non-local properties of surface layers in the presence of elliptical stress concentrators.

Design/methodology/approach: Using the proposed approaches of developed mathematical modelling and open software for calculating FEniCS, it were established the most dangerous angles of stress concentrator orientation and the required thickness of the hardened zones of parts, which ensures their minimum softening during operation.

Findings: It is shown that for an elliptical stress concentrator with any orientation angle, there is a certain key size of surface hardening thickness, the exceeding the value of which does not have influence on the operational strength of the parts, but rise the price of technological operations.

Research limitations/implications: In this paper proposes a method for computation the impact of the orientation of the surface elliptical stress concentrators on the contact strength of parts under conditions of dominate friction power loads.

Practical implications: The obtained results were used to set the modes of plasma hardening, which increase the contact strength of railway parts with elliptical stress concentrators.

Originality/value: Using the approaches of contact mechanics, mathematical and computer modelling, methods of controlling the contact strength of the parts with the surface elliptical stress concentrators were proposed for the first time.

Keywords: Elliptical stress concentrators, Optimization of hardening modes, Power loads, Computational mechanics

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METHODOLOGY OF RESEARCH, ANALYSIS AND MODELLING



1. Introduction

The most important feature of modern transport systems functioning is the requirement regarding their high reliability under operating conditions [1]. Particularly important is this problem in the case of rolling stock undercarriage, since the safety of transport infrastructure depends on the reliability of their operation [2].

The wear and tear and the destruction of components under intensive loads, in most cases, initiates just at the surface. Therefore, the condition of surface layers is a constraining factor, which determines the serviceability of a component. That is why the most effective method of solving the problem of increasing the reliability of components of mechanical systems is ensuring the specified structural and energetic state of the surface layers of components, taking into account specific operating conditions [3].

A great number of theoretical and application-focused works [1-4] are devoted to the problem of employing surface engineering approaches to obtain elements with a predetermined lifecycle. They suggest approaches for establishing optimal regimes of surface hardening when using different technologies in order to achieve the desired properties of surface layers of critical items.

In most cases, technological solutions refer to components which surface layers do not suffer from geometric stress concentrators, are characterised by a certain degree of surface roughness and lack physical non-locality.

One of the characteristic features of transport engineering components is the potential presence of adjacent to the surface layers geometric stress concentrators, which can be of structural, technological or operational origin. Their presence significantly changes the stress-strain state of components and inevitably affects their serviceability. Therefore, it is important to be able to "compensate" for the presence of the concentrators to minimize their impact on the performance of the components.

Hardening of components subjected to stress concentrators is a far more complicated process, requires novel approaches, and is scarcely represented in the literature. The solution is possible taking into account the non-locality and geometry of items, the structural-energetic state of surfaces when using functional-gradient models of continuum mechanics, as well as methods of nonlinear functional analysis, which substantially complicates both the statement and the search for a solution of such problem types [5,6].

The optimization of surface hardening techniques for the most common circular stress concentrators is discussed in [7]. Optimal hardness penetration depths ensuring the maximum possible structure serviceability have been

established in the presence of circular defects having different diameters and distances from the interface.

The analysis of geometrical and spatial characteristics of stress concentrators, which may be present in components of mechanical items, suggests that they can have shapes, including those other than circular, in particular an elliptical ones.

The influence of elliptical stress concentrator on the serviceability of components with structurally heterogeneous material state is practically unrepresented in the modern literature. That is why is very important to establish the role of spatial location and orientation of the elliptic-type subsurface stress concentrator in the formation of performance characteristics of functionally graded structures. Similar studies are vital for optimising the stress-strain state of a component in terms of the criteria for providing a given level of reliability by surface engineering methods.

2. Problem formulation of establishing the operating parameters of functionally graded structures within the presence of elliptical stress concentrators

The use of mathematical and computer modelling is one of the most effective approaches for analysing the parameters of structures during their operation [8,9], its implementation makes empowers determining the strength parameters of components with sufficient practical accuracy and avoiding cost-intensive experimentation.

In this paper we consider the formulation of the problem in the following generalised two-dimensional formulation:

1. Let the study body (component) occupy an area of space, let the surface of the body be denoted by ∂X .
2. In the body let set areas Y_1, \dots, Y_n , that are elliptical stress concentrators. Each area Y_i ($i = 1, \dots, n$) is characterised by the following parameters (R_1^i, R_2^i, α^i) , where R_1^i, R_2^i are radii of the main axes, α^i is an angle of rotation of elliptical concentrators relative to an axis perpendicular to the plane of the study object (Fig. 1).
3. On the surface of a part ∂X let us specify the set of sub-areas $\partial X^* = \bigcup_{i=1}^m \partial X_1^i$, in which a force load has the form of $\vec{F}_1^i = \vec{F}_1^i(x)$, $x \in \partial X$.
4. When considering the behaviour of an item under contact loading, we use the non-local mathematical model of an elastic continuum given in [10].
5. As a result of force loads, a stress state is formed in the component, which is characterised by the tensor $\hat{\sigma} = \hat{\sigma}(x)$, $x \in X$.

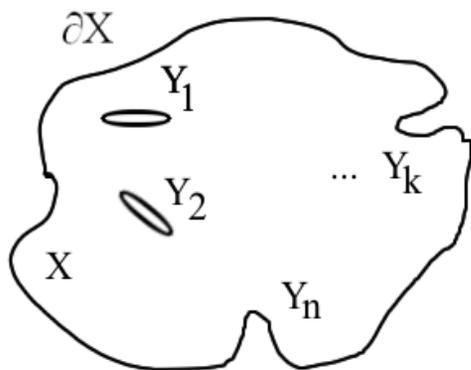


Fig. 1. Schematic representation of a study object element: X is an area occupied by the body, ∂X is body boundaries, Y_1, \dots, Y_n are elliptical stress concentrators.

6. Since the rated stresses do not unambiguously determine the operating parameters of structures [11], for the contact strength analysis we use expression:

$$k(x) = 1 - \frac{\sigma_m(x)}{\sigma^*(x)}, \tag{1}$$

where $k(x)$ is the contact strength safety factor, $\sigma_m(x)$ is the stress tensor equivalent representation by von Mises, $\sigma^*(x)$ is the material strength properties at the point x .

- 7. The value of $k(x)$ changes within the interval $-\infty; 1$, at $k(x) = 1$ no softening is observed, at $k(x) \leq 0$ softening has occurred.
- 8. For the purpose of construction analysis, let us take the total relative volume of material with a contact strength value below zero as a functional that describes the basic

performance characteristic of the item, i. e. the level of softening [5, 7]:

$$L = \frac{\dim(k(x) \leq 0)}{\dim(x \in X)}, \tag{2}$$

where $\dim(\dots)$ – the size of the area that satisfies the specified conditions.

As can be seen from relation (2), the functional L operates not with absolute, but with relative values of structural softening, and is dimensionless.

In general terms, the functional L depends on loads, the structural-energy state distribution in the item and on stress concentrators.

Let us examine the dependence of the functional L on the parameters Y_i ($i = 1, \dots, n$), i.e. $L = L(Y_1, \dots, Y_n) = L(Y_i)$ or $L = L(R_1^i, R_2^i, \alpha^i)$, i.e. let us set such parameters (R_1^i, R_2^i, α^i) , which minimise the functional (2).

We use the FEniCS finite element analysis package and its Python implementation [12] to calculate and find the optimum operating parameters of the study structure, and the Matplotlib package [13] for post-processing (displaying) the results.

3. Case study of determining of elliptical defect orientation in surface layers of items with non-local mechanical characteristics

As a simulation example, let us consider a 2-dimensional contact problem involving a body with gradient mechanical characteristics that contains a single subsurface elliptical stress concentrator (Fig. 2).

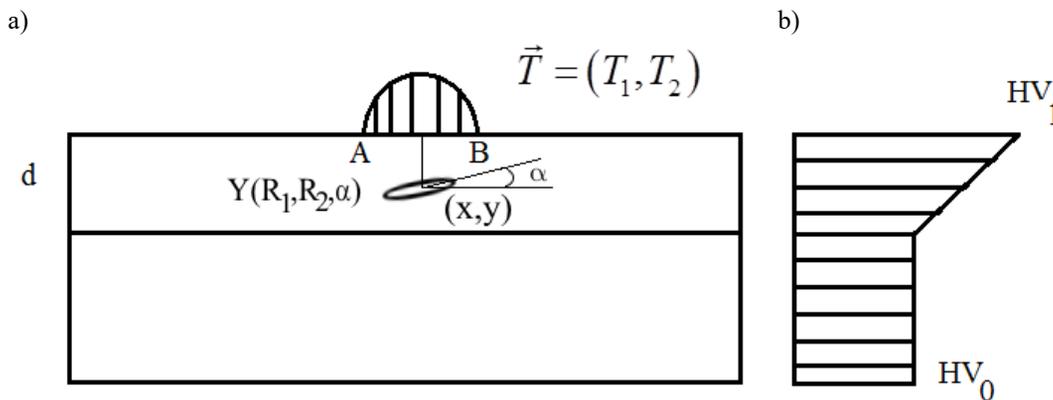


Fig. 2. Formulation of a 2-dimensional contact state analysis problem: a) the study object: AB is a contact patch (area of contact interaction), $\vec{T} = (T_1, T_2)$ is the load vector, d is the hardening zone thickness, R_1, R_2 are main radii of elliptic stress concentrator, α is the concentrator angle of rotation concerning the normal to the load plane relation, (x, y) are coordinates of the elliptical stress concentrator centre; b) micro-hardness depth distribution epiurus: HV_1 is a surface micro-hardness value, HV_0 is a micro-hardness value regarding the material depth

Note that this formulation of the contact problem is classical for the analysis of the operational parameters of the investigated parts [14,15].

For direct estimations of the level of softening let us take as example that $HV_1 = 3.900$ MPa, $HV_0 = 2.600$ MPa, $d = 2$ mm, the initial force load value is $12.5 \cdot 10^4$ N, friction coefficient is 0.25 [5], $R_1 = 4$ mm, $R_2 = 2$ mm, $\alpha = 30^\circ$, the ellipse centre is at 5 mm depth from the centre of contact interaction area.

Initial data for estimations are typical for locomotive wheel-rail contact pairs. For the transition from microhardness values to mechanical property parameters, we use findings presented in [16].

According to the results of this work, for engineering calculations, it is possible to accept the presence of a linear relationship between microhardness and elastic modulus ($E \approx \alpha \cdot HV$, where $\alpha = 0.013$ for steels). We will take the value of Poisson's ratio equal to 0.25, since its value depends less on the change in microhardness.

The resultant stress field is shown in Figure 3.



Fig. 3. Stress distribution by von Mises in the study structure within the presence of an elliptical stress concentrator, 10^9 Pa

The stress state analysis (Fig. 3) indicates that the maximum level of stress develops directly in the area of contact interaction and near the stress concentrator, which significantly changes the stress field as compared to the area with no stress concentrator in place.

Since the stressed state does not explicitly determine the strength properties of an object [14], to characterize its performance properties we analyse the structural softening based on the relation (1) taken from [11,17].

The distribution of softening values (Fig. 4) shows that the most significant loss of strength properties of the material is observed in the zone of stress concentrator and directly on the contact interaction surface.

In the general case, the stress-strain state of the structure is influenced by the dimensions of the elliptical concentrator, its geometric location (location from the surface), the ratio of the principal radii, and the angle of rotation.

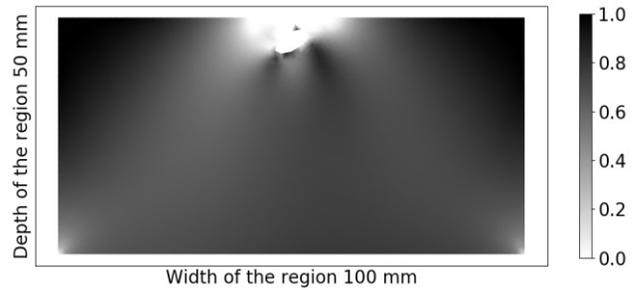


Fig. 4. The distribution of stress-strain magnitude in a body within the presence of an elliptical stress concentrator

Let us investigate the dependence of elliptic stress concentrator rotation angle on structural loosening, namely, function (2) on the parameter α at different values of force loads $10 \cdot 10^4$ N, $12.5 \cdot 10^4$ N, $15 \cdot 10^4$ N, and friction coefficients 0, 0.125, 0.25, 0.5.

The angle α varies from 0° to 180° in increments of 1° , i.e., 180 numerical experiments were conducted for each value of friction coefficient at given load.

As a result, the following dependence of softening zone level on the angle α is produced (Fig. 5).

The resulting regularities indicate that, regardless of the absolute value of the contact loads and friction coefficient, the values of angle α close to 20° and 110° are the least safe for operation and cause the maximum amount of softening.

An analysis of the obtained results shows that the operational strength parameters depend not only on the location of the elliptical stress concentrator and its dimensional characteristics, but also on the geometric orientation with respect to external loads. The latter factor is often predominant in assessing the performance of rolling stock wheels [18].

A statistical analysis of the formation of defects in railway wheelsets also indicates a significant role of oriented subsurface stress concentrators when they fail during operation. According to open literature sources, such stress concentrators can be the cause of defects in more than 40% of cases [19].

4. Substantiation of practical recommendations for increasing operational parameters of structural elements with elliptical stress concentrators

In the practice of railway transport systems operation, plasma hardening is used to improve the durability of locomotive wheels. The most important characteristic of this

technology is the hardened layer depth, which allows providing the maximum increase in reliability parameters of an item. In [20], a process mechanics methodology was proposed to justify the depth of surface layer hardening depending on the operating conditions in the absence of stress concentrators.

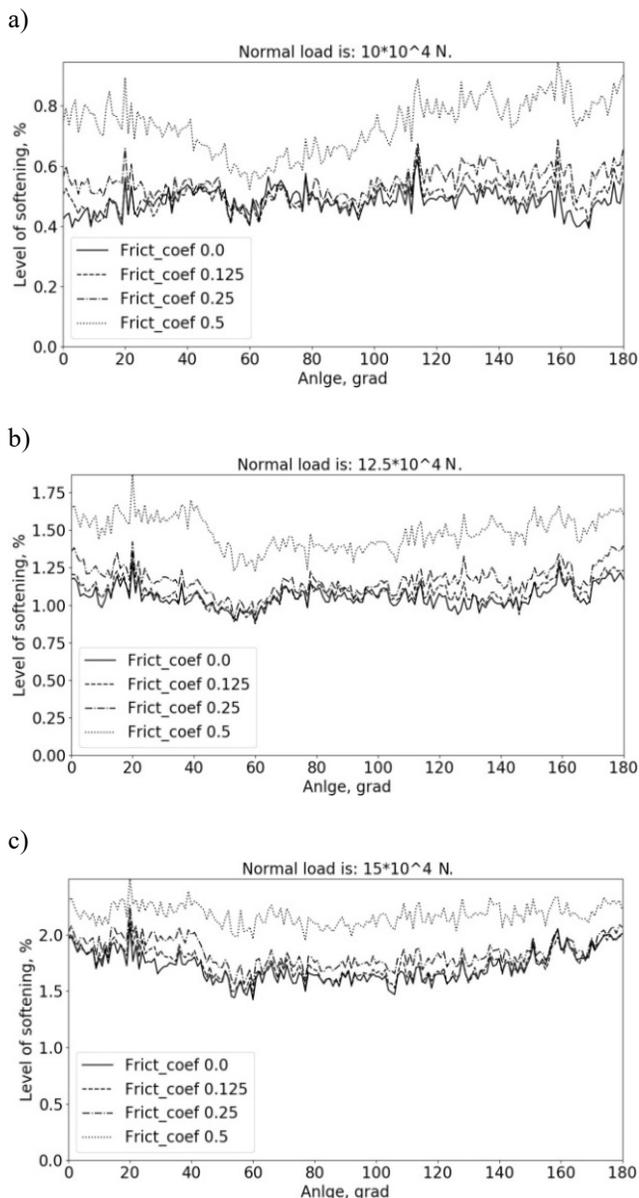


Fig. 5. Dependence of the softening level L on the elliptical stress concentrator rotation angle α under the normal load: a) $10 \cdot 10^4$ N, b) $12.5 \cdot 10^4$ N, c) $15 \cdot 10^4$ N

Given the presence of stress concentrators in the surface layers, the operating conditions change and, as a

result, a necessity to modify the surface hardening regimes arises [21].

Therefore, as an example, we make a computational justification and determine the optimum depth of surface hardening for a component that contains an elliptical stress concentrator at a depth of 5 mm from the surface with the principal radii of 4 mm and 2 mm and an angle of rotation relative to the axis of 20° .

The hardening depth d varies from 0 to 45 mm in increments of 0.5 mm.

As a result, at the normal load of $12.5 \cdot 10^4$ N and friction coefficient of 0.125 we receive the following graphical dependence of softening level magnitude on the hardening depth d (Fig. 6).

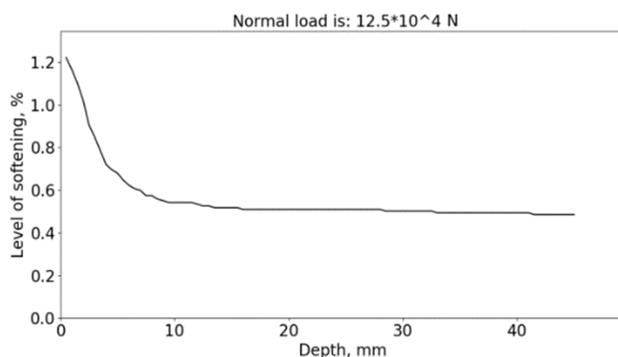


Fig. 6. Dependence of softened zone magnitude on technological modification depth d within presence of elliptic stress concentrator under the normal load $12.5 \cdot 10^4$ N and friction coefficient 0.125

As is illustrated by the results in Figure 6, there is a certain value of the hardened layer d^* at which softening parameter does not change significantly. In this case $d^*=15$ mm.

Since on the surface of the hardened part the microhardness value is 3900 MPa, and at a depth of 15 mm – 2600 MPa (initial value), we need to obtain a surface of railway wheel with a microhardness gradient with the level

$$g = \frac{3900 - 2600}{0.015} = 8.7 \cdot 10^{10} \frac{Pa}{m}$$

The given method allows estimating the optimum depth of technological hardening and establishing that it is not feasible to perform technological hardening deeper, since with the increase in energy consumption required for hardening the operational properties of an item change insignificantly, which allows increasing the durability parameters of rolling stock wheels to the greatest extent [22].

5. Conclusions

1. The problem of optimisation of operational parameters of items containing elliptic stress concentrators is considered on the basis of modern computational and applied mechanics approaches.
2. Using FEniCS package and its implementation in Python language, the stress state has been analysed using a locomotive wheel as an example under the load regimes close to the operating ones and the most dangerous angles of stress concentrator arrangement which are approximately equal to 20° and 110° have been determined.
3. It was established that, depending on the stress concentrator location, there exists a limit value of the hardened layer depth, at which further increase of contact strength parameters does not change significantly.
4. For the presented operating conditions of the railway wheel are recommended such plasma hardening modes, that provide the value of the microhardness gradient of the surface layers at the level $8.7 \cdot 10^{10}$ Pa/m, which further leads to the maximum extension of the life cycle.

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