International Journal of Thermofluids 27 (2025) 101199

Contents lists available at ScienceDirect

International Journal of Thermofluids

journal homepage: www.sciencedirect.com/journal/international-journal-of-thermofluids

A mathematical model for the determination of the parameters of a gas in an open thermodynamic system in contact with the environment

Serhii M. Ponomarenko [a], Oleksandr V. Zhevzhyk [b], Iryna Yu Potapchuk [b],
Liudmyla B. Kabakova [a], Dmytro O. Yelatontsev [a,*]

[a] *M.S. Poliakov Institute of Geotechnical Mechanics of the National Academy of Sciences of Ukraine, 2-a Simferopolska St., Dnipro, Dnipropetrovsk Oblast, Ukraine*
[b] *Ukrainian State University of Science and Technologies, 2 Lazaryana St., Dnipro, Dnipropetrovsk Oblast, Ukraine*

ARTICLE INFO

Keywords:
Mathematical modelling
Thermodynamic parameters
Gas leakage
Heat transfer
Compressed air

ABSTRACT

Leaks in gaseous storage and supply systems impose security and economic threats that are influenced by the heat exchange. A methodology is proposed to assess thermodynamic changes in a leaky system utilizing a mathematical model for the "pipe-hole" scenario based on the Poiseuille equation for laminar gas leakage with a capillary diameter below 0.1 m. Differential equations are derived to analyze the time-dependent thermodynamic variations of compressed air in the “pipe-hole” system via the energy balance equation. A mathematical model was developed for general case studies of open thermodynamic variable-mass gas systems. The FORTRAN software tool was used to solve the system of differential equations using the Runge-Kutta method, with four orders of accuracy. A specific examination was conducted on a thermodynamic system that lacks a constant gas supply and stationary convective heat exchange in a single-layer gas container with the environment. The ranges of the control parameter variation were considered as follows: air pressure p_0 from 150 kPa to 300 kPa and temperature T_a from 15 °C to 25 °C. This study reveals how the density, pressure, and temperature of compressed air vary over time owing to leakage and heat fluxes. It was determined that as initial air pressure decreases from $p_0 = 150$ kPa, the impact of heat flow rates markedly surpasses that of air leakage rates. A potential application of this research could be the software for monitoring devices for high-pressure systems designed for the storage and distribution of gaseous energy carriers.

Abbreviation

C_p	heat capacity at constant pressure, J·kg^{-1}·K^{-1};
C_V	heat capacity at constant volume, J·kg^{-1}·K^{-1};
F	surface area, m^2;
G_{ex}	mass flow rate of uncontrolled gas leakage, kg·s^{-1};
G_{in}	mass flow rate of gas entering the system, kg·s^{-1};
G_{out}	mass flow rate of controlled gas leakage, kg·s^{-1};
i_{ex}	enthalpy of uncontrolled gas leakage, J·kg^{-1};
i_{in}	enthalpy of the gas entering the system, J·kg^{-1};
i_{out}	enthalpy of controlled gas leakage, J·kg^{-1};
$K_{\lambda\alpha}$	heat transfer coefficient, W·m^{-2}·K^{-1};
k	adiabatic index;
p	pressure (absolute), Pa;
p_a	atmospheric pressure, Pa;
Q	thermal energy, J;
R	gas constant, J·kg^{-1}·K^{-1};
T	temperature, K;
T_a	ambient temperature, K;
T_{in}	temperature of the gas entering the system, K;
V	volume, m^3;
V_0	initial volume, m^3;
U	internal energy, J;
α_1	heat transfer coefficients from the internal side of the system, W·m^{-2}·K^{-1};
α_2	heat transfer coefficients from the outer side of the system, W·m^{-2}·K^{-1};
δ	average wall thickness of the system, m;
λ	coefficient of heat conductivity of the system wall, W·m^{-1}·K^{-1};
μ	coefficient of kinematic viscosity, m^2·s^{-1};
ρ	density, kg·m^{-3};
τ	time, s.

---

* Corresponding author.
E-mail address: iambenadar@proton.me (D.O. Yelatontsev).

https://doi.org/10.1016/j.ijft.2025.101199

1. Introduction

The application of compressed air and its storage units makes pneumatic energy the fourth resource after electricity, natural gas, and water. In the industrial sector, this energy source is used in a wide range of pneumatic tools and plays a crucial role in many nonmanufacturing sectors. In particular, transportation, construction, mining, agriculture, recreation, and services are active compressed air consumers. Pneumatic energy is also used in various technological operations such as aeration, filtration, cryogenics, oxidation, fractionation, cooling, and dehydration [1–3]. Recently, the thermal destruction of various materials, where compressed air is used as a high-temperature coolant in thermal tools, has become increasingly attractive [4].

Despite the obvious attractiveness of pneumatic energy, it has a rather high cost, and it is necessary to consider the overall efficiency of the air supply system for the final consumption [5]. The efficiency and reliability of a pneumatic energy consumer's operation depend on the quantity and quality of the compressed air, namely, the air pressure and flow rate. These parameters are directly affected by two factors: the presence of compressed air leakage in the air supply system and its heat exchange with the environment.

Compressed air leakage is the largest source of energy loss in industrial systems of pneumatic energy production, supply, and consumption. According to the United States Department of Commerce, a typical industrial enterprise uses 10 % electricity to operate pneumatic energy systems, and 20 % to 30 % of compressed air is lost owing to leaks. This causes nonproductive electricity losses in air supply systems. The need to increase energy consumption also leads to a 20–25 % increase in greenhouse gas emissions [6].

Many studies have been devoted to increasing the efficiency of compressed air use by determining leakages and their magnitudes. For example, [7] proposed a methodology for a local comparative analysis using the correlation between compressed air supply and air production. This methodology determines the potential for reducing compressed air losses and is aimed at improving the air supply system to increase the efficiency of its use. The Ontario Mining Association launched a Compressed Air Leakage Management project to save energy by preventing leaks in the compressed air supply systems. The purpose is to conduct a comprehensive audit of electricity use and energy-saving opportunities in compressed air systems based on the readings of flow and pressure measuring devices in different types of underground mines [8].

Classical thermodynamics deals with the most general properties of macroscopic systems, including energy transmission and conversion. This research primarily focuses on studying the thermodynamic parameters of systems with a very large number of particles, based on the generalization of experimental data. In this section of phenomenological science, the heat exchange of thermodynamic systems, both closed and open, with the environment was studied in detail [9–11].

The basic provisions of thermodynamics are three fundamental laws obtained empirically: the zero law (related to the concept of thermal equilibrium), first law (on the conservation of energy), and second law (in the direction of thermodynamic processes). The subject of research in thermodynamics is the distribution of heat and energy in three types of thermodynamic systems depending on their connection with the environment [11–13]:

1) isolated (no exchange of mass or energy with the environment);
2) closed (without mass exchange with the environment, but with redistribution of heat energy);
3) open (there is an exchange of mass or energy with the environment).

Non-hermetic systems for storing and transporting compressed air to pneumatic energy consumers are the third type of thermodynamic systems. It is a well-known practice to apply the first law of thermodynamics to this type of system, where energy conservation is determined based on the consideration of simple cases of steady-state and steady flow with the addition of details and complexity of the physical processes under study [12]. This approach to consider the effect of heat exchange with the environment of a leaky object on the value of gaseous medium leakage from a leaky object was used in [14,15].

Many scientists have studied the problems of assessing natural gas leakage from onshore [16–18] and subsea [19] gas transmission and distribution pipelines. They are united by a common approach to determining the gas leakage rate, which is an important parameter for the risk assessment and analysis of gas pipeline failures. These works focused on the gas-dynamic parameters of the gas flow, considering the local gas leakage, which creates many economic and social problems.

Consequently, the objective of this research was to conduct an analytical examination of the regularities pertaining to two autonomous phenomena: the laminar flow regime of a gas medium egress (microleakage) from a defective entity and its thermal interaction with the surrounding environment. The object of the present study is the physical processes in an open thermodynamic system with gas mass change. The novelty of the study involves the development of a mathematical model of changes in the thermodynamic parameters of gas in a system with heat exchange with the environment and simulated leakage, determining the rate of gas leakage from a system of constant volume.

2. Theoretical

The calculation methodology consists of a general mathematical model of the time-varying thermodynamic parameters of a gas that occurs in a leaky volume with rigid walls, which is under the influence of heat exchange with the environment, and testing of this model for a specific case with an element of the compressed air supply network. The long-term wear of the system components is accounted for in the model by the wall thickness, which can change over time owing to natural corrosion.

2.1. Mathematical model

The systems for the storage and transportation of gaseous substances are diverse, depending on their technological applications. Leaky areas in storage units make it very difficult to determine the total leakage rate, and the geometric characteristics of the system and its heat exchange with the environment must also be considered. Considering the multifactorial nature of the processes in an open thermodynamic system, it is generally accepted to consider its simplified model in the form of a "pipe-hole".

It is generally accepted that mass-averaged approximation is used to describe the gas dynamic processes. It is assumed that the change in gas parameters in the vessel occurs only with time and is determined by the conditions at the inlet and outlet elements of the vessel and by the processes of heat and mass transfer in it, including its surface. In this case, when studying the processes in a vessel, it is acceptable to use mathematical models based on a system of ordinary differential equations with respect to gas parameters, which are a function of time and do not depend on spatial coordinates [16–19].

For an open thermodynamic system with a free volume V and gas pressure p, which is influenced by heat exchange (Q) with the

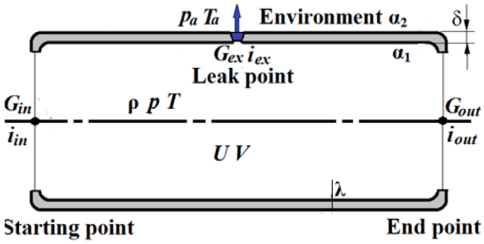

Fig. 1. Open thermodynamic system with a variable gas mass.

environment (Fig. 1), we apply the provisions of the first law of thermodynamics.

Following the law of conservation of energy, the heat balance equation for the system in the differential form from τ is as follows:

$$G_{in}i_{in}d\tau+dQ=dU+pdV+G_{out}i_{out}d\tau+\ G_{ex}i_{ex}d\tau. \tag{1}$$

Solving Eq. (1) concerning the change in the internal energy of a gas in a volume with rigid walls ($V=V_0=const;\ dV/d\tau=0$) can be represented by the following equation:

$$\frac{dU}{d\tau}=G_{in}i_{in}+\frac{dQ}{d\tau}-G_{out}i_{out}-G_{ex}i_{ex}. \tag{2}$$

The internal energy of a gas through the adiabatic index k is defined as:

$$U=\frac{pV}{(k-1)}. \tag{3}$$

The time derivative in Eq. (3) for $dV/d\tau=0$ is as follows:

$$\frac{dU}{d\tau}=\frac{\left[\frac{Vdp}{d\tau}-\frac{pVdk}{(k-1)d\tau}\right]}{(k-1)}. \tag{4}$$

From a joint consideration of Eqs. (2) and (4) under the condition *that* $V=V_0=const$, we have

$$(k-1)\left(G_{in}i_{in}+\frac{dQ}{d\tau}-G_{out}i_{out}-G_{ex}i_{ex}\right)=\frac{V_0dp}{d\tau}-\frac{pV_0dk}{(k-1)d\tau}.$$

Thus,

$$\frac{dp}{d\tau}=\frac{(k-1)}{V}\left[G_{in}i_{in}-G_{out}i_{out}-G_{ex}i_{ex}+\frac{dQ}{d\tau}+\frac{pV}{(k-1)^2}\frac{dk}{d\tau}\right]. \tag{5}$$

In most cases, the Newton-Richman law is used to calculate the heat flux density (Q), where the heat flux is a formal expression. In this expression, all factors that affect the intensity of heat transfer are not explicitly considered. This makes it necessary to use the heat transfer coefficient, which has different values for individual sections of the contact surface owing to the uneven intensity of the heat exchange over the entire contact surface. To determine the total heat transfer coefficient, we used the convective heat transfer coefficients between the gas and system wall and the system wall and the environment. Then

$$Q=-\frac{(T-T_a)F\tau}{\left(\frac{1}{\alpha_1}+\frac{\delta}{\lambda}+\frac{1}{\alpha_2}\right)}=-K_{\alpha\lambda}(T-T_a)F\tau, \tag{6}$$

where $K_{\alpha\lambda}=(1/\alpha_1+\delta/\lambda+1/\alpha_2)^{-1}$ is the heat transfer coefficient.

The assumption of a constant ambient temperature T_a is made for relatively short time intervals, during which real temperature fluctuations can be neglected. The temperature was assumed to be averaged over time as the time interval increased.

In heat transfer dynamics, the heat transfer coefficient generally depends on the structure, density, humidity, pressure, and temperature of the container material. The combination of these factors causes certain difficulties in determining the heat transfer coefficient, which must be determined by a special study of the material used. Therefore, in technical calculations, reference values of the thermal conductivity coefficient were considered, taking into account the physical characteristics of the material.

The adiabatic index is a function of temperature $k=f(T)$, and its time derivative is:

$$\frac{dk}{d\tau}=\frac{dk}{dT}\frac{dT}{d\tau}. \tag{7}$$

Assuming $i_{out}=i_{ex}=C_pT$ and taking into account Eqs. (6) and (7), equality (5) for $V=V_0$ can be represented as:

$$\frac{dp}{d\tau}=\frac{(k-1)}{V_0}\left[G_{in}i_{in}-(G_{out}+G_{ex})C_pT-K_{\alpha\lambda}(T-T_a)F+\frac{pV_0}{(k-1)^2}\frac{dk}{dT}\frac{dT}{d\tau}\right]. \tag{8}$$

Differentiating the state equation $p=\rho RT$ in time, we have

$$\frac{dp}{d\tau}=\rho R\frac{dT}{d\tau}+RT\frac{d\rho}{d\tau}. \tag{9}$$

For variable heat transfer coefficients, when their components depend on the temperature, Eq. (9) is generalized by introducing appropriate empirical relationships. The resulting nonlinearity of the equations can be easily overcome using an iterative calculation procedure.

To determine the gas mass leakage from the system in the laminar regime, the Poiseuille formula was used. Thus, we derive the following relationship, Eq. (10):

$$G_{ex}=\frac{d(\rho V_0)}{d\tau}=\frac{\pi d_0^4\rho}{128\mu\delta}(p-p_a)\Rightarrow\ \frac{d\rho}{d\tau}=-\frac{G_{ex}}{V_0}=-\frac{\pi d_0^4\rho}{128V_0\mu\delta}(p-p_a). \tag{10}$$

The use of Poiseuille's formula is justified for the laminar regime of gaseous substance leakage from a thermodynamic system under overpressure and rigid walls with a constant degree of leakage. It is also true that:

- gas leaks through leaky parts of the system with low leakage rates;
- the geometric parameters of the total leakage points are constant and can be modelled by the capillary diameter.

The application of the Poiseuille equation limits the application of the model to laminar leakage flows, the gas dynamics of which differ significantly from other gas leakage regimes, especially from the regime of developed turbulence.

By introducing turbulence in the transition from the laminar (viscous) leakage regime (small diameters of the leakage orifice) to the molecular-viscous regime instead of the Poiseuille formula, a more complex general Knudsen formula can be applied to calculate the flow in Eq. (10). For the turbulent regime, dependencies based on the Bernoulli equation and empirical values of the flow coefficient are applied.

Taking into account relationships from Eq. (10), the Eq. (9) takes the form

$$\frac{dp}{d\tau}=\rho R\frac{dT}{d\tau}-RT\frac{G_{ex}}{V_0}. \tag{11}$$

From a joint consideration of Eqs. (8) and (11) taking into account Mayer's formula (C_p– $C_V=R$) and the fact that $C_p/\ C_V=k$, it follows:

$$\frac{dT}{d\tau}=\frac{1}{V_0\left(\rho R-\frac{p}{k-1}\frac{dk}{dT}\right)}\left\{(k-1)\left[G_{in}i_{in}-\frac{(G_{out}+G_{ex})kRT}{k-1}-K_{\alpha\lambda}(T-T_a)F\right]+RTG_{ex}\right\}. \tag{12}$$

Taking into account Eqs. (10) and (12), Eq. (9) has the following form:

$$\frac{dp}{d\tau}=\frac{\rho R}{V_0\left(\rho R-\frac{p}{k-1}\frac{dk}{dT}\right)}\left\{(k-1)\left[G_{in}i_{in}-\frac{(G_{out}+G_{ex})kRT}{k-1}-K_{\alpha\lambda}(T-T_a)F\right]+RTG_{ex}\right\}-RT\frac{G_{ex}}{V_0}. \tag{13}$$

Generally, the system of Eqs. (10), (12), and (13) characterizes the change in the thermodynamic parameters of the gas in an open thermodynamic system with a change in the gas mass. In practice, three variants of the physical processes in this system are possible:

- first – the one considered above;
- second – at $dk/dT = 0$;
- third – at $G_{in} = G_{out} = 0$.

In the second variant, using the above methodology for defining the differential equations of the time-varying temperature and gas pressure in the system, we have

$$\frac{dT}{d\tau} = \frac{1}{\rho R V_0}\left\{(k-1)\left[G_{in}i_{in} - \frac{(G_{out}+G_{ex})kRT}{k-1} - K_{\alpha\lambda}(T-T_a)F\right] + RTG_{ex}\right\}. \tag{14}$$

$$\frac{dp}{d\tau} = \frac{1}{V_0}\left\{(k-1)\left[G_{in}i_{in} - \frac{(G_{out}+G_{ex})kRT}{k-1} - K_{\alpha\lambda}(T-T_a)F\right] + RTG_{ex}\right\} - RT\frac{G_{ex}}{V_0}. \tag{15}$$

In the third variant, the same methodology yields the following defining equation:

$$\frac{dT}{d\tau} = -\frac{(k-1)[RTG_{ex} + K_{\alpha\lambda}(T-T_a)F]}{V_0\left(\rho R - \frac{p}{k-1}\frac{dk}{dT}\right)}. \tag{16}$$

$$\frac{dp}{d\tau} = -\frac{R}{V_0}\times\left\{\frac{\rho(k-1)[G_{ex}RT + K_{\alpha\lambda}(T-T_a)F]}{\rho R - \frac{p}{k-1}\frac{dk}{dT}} + TG_{ex}\right\}. \tag{17}$$

A block diagram of the developed methodology is shown in Fig. 2.

In the theory and practice of non-destructive control of various objects, many methods have specific features. One of them is the manometric method of control or the "pressure drop" method. The degree of leakage of an object σ refers to the design characteristics of the object itself, and its connection with the liquid or gas inside it is determined by the leakage rate of this medium for σ. When applying the method of pressure drop in a closed volume, the leakage rate (degree of leakage σ) is defined as the product of the volume and the pressure change per unit time [20–22]. For a system with constant volume, the following relationship is valid:

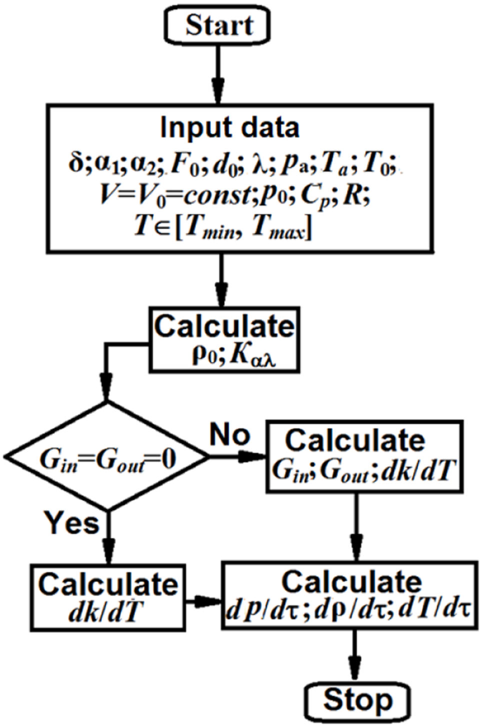

Fig. 2. Block diagram of the methodology: $T_{\min}$ and $T_{\max}$ are the ranges of temperature change.

$$\frac{d\sigma}{d\tau} = \frac{d(pV)}{d\tau} = V_0\frac{dp}{d\tau}. \tag{18}$$

Considering equality (18), Eqs. (13), (15), and (17) will have the following form:

$$\frac{d\sigma}{d\tau} = \frac{\rho R}{\rho R - \frac{p}{k-1}\frac{dk}{dT}}\left\{(k-1)\left[G_{in}i_{in} - \frac{(G_{out}+G_{ex})kRT}{k-1} - K_{\alpha\lambda}(T-T_a)F\right] + RTG_{ex}\right\} - RT\frac{G_{ex}}{V_0}. \tag{19}$$

$$\frac{d\sigma}{d\tau} = (k-1)\left[G_{in}i_{in} - \frac{(G_{out}+G_{ex})kRT}{k-1} - K_{\alpha\lambda}(T-T_a)F\right] + RTG_{ex} - RT\frac{G_{ex}}{V_0}. \tag{20}$$

$$\frac{d\sigma}{d\tau} = -R\times\left\{\frac{\rho(k-1)[G_{ex}RT + K_{\alpha\lambda}(T-T_a)F]}{\rho R - \frac{p}{k-1}\frac{dk}{dT}} + TG_{ex}\right\}. \tag{21}$$

Optimization methods for gas leakage minimization based on this model are not currently available, and their development requires further research.

The above methodology was developed for a single-layer (homogeneous) container system with thickness δ, which after some adjustments can be applied to a multilayer wall. In this case, the total thermal resistance of a multilayer wall consisting of several heterogeneous and closely adjoining layers with thicknesses and thermal conductivity coefficients is equal to the sum of partial thermal resistances. The long-term deterioration of the system components is accounted for in the model by the wall thickness, which undergoes natural corrosion over time.

2.2. Calculation methods

The Runge-Kutta method of the 4th order of accuracy was used to solve the system of differential equations, in which the mass of gas remaining in the tank and lost per unit of time (instantaneous mass flow) was determined at each time step. Considering that, in general, the resulting system of equations is inhomogeneous and nonlinear, this numerical method was selected. The error was controlled by double the calculation of the values with a halved time step, and the state equation accuracy was also monitored at each step.

It is not possible to compare the model with experimental data for large systems at the moment because of their absence owing to the impossibility of providing around the system a stationary uniform temperature field, constancy of the ambient pressure, heat transfer, and a number of other factors.

To validate the mathematical model, a special case of supplying compressed air to consumers is considered. The total leakage of the system is modelled by the diameter of the hole through which the air leaks in the laminar mode. The initial parameters of the air-supply system used in the calculations are listed in Table 1.

The initial density of the compressed air is determined by the state equation. The dependence of the dynamic viscosity coefficient as a function of temperature $f(T)$ is approximated by a polynomial of known reference data in the temperature range $223.15\text{ K} \le T \le 423.15\text{ K}$. The numerical value μ was calculated according to the obtained dependence for a given temperature at each time interval.

3. Results and discussion

Under normal conditions, the physical and chemical properties of

Table 1
Initial parameters of the air supply system.

Constant	Amount	Unit
V_0	0.0025	m^3
δ	10^{-3}	m
α_1	10	$W{\cdot}m^{-2}{\cdot}K^{-1}$
α_2	5	$W{\cdot}m^{-2}{\cdot}K^{-1}$
$F_0 = F$	0.0025	m^2
$d_0 = d$	$3{\cdot}10^{-5}$	m
λ	45	W/(m·K)
p_a	10^5	Pa
T_a	25; 20; 15	°C
$T_0 = T$	293.15	K
$p_0 = p$	300; 200; 150	kPa
C_p	1007	$J{\cdot}kg^{-1}{\cdot}K^{-1}$
k	1.4	
R	8.314	$J{\cdot}kg^{-1}{\cdot}K^{-1}$

gases and their mixtures differ significantly. The diffusion properties of different gases exhibit large differences, which significantly affect their safety during transportation and storage. Systems and their individual elements for gaseous substances by their nature cannot be absolutely airtight, which also increases the risk of possible gas losses. The uncertainty in determining the location and magnitude of the local (local) leakage of a gas object makes it necessary to estimate the gas losses (total leakage) from the total leakage of the object. Compressed air is the most common gaseous substance (a mixture of gases) and is widely used in the industry for energy storage and transmission. However, the high energy intensity of its production requires a constant search for solutions to reduce the nonproductive losses of this energy carrier. Therefore, this study focuses on theoretical calculations of air supply problems.

The results of the calculations of the time variation of the compressed-air thermodynamic parameters in an open thermodynamic system for the initial parameters given in Table 1 are shown in Fig. 3.

A simplified version of the physical process with $G_{in} = G_{out} = 0$ and calculation of dk/dT in the temperature range $T_{min} = 223.15\ K \le T \le 423.15\ \text{K} = T_{max}$ (Fig. 2) was considered. During the calculations, the ambient temperature and compressed air pressure were varied in accordance with the data in Table 1.

The results of the calculations show that changes in the time of the thermodynamic parameters of compressed air in the “pipe-hole” model (Fig. 3) indicate the following regularities:

1) The air pressure and air density decreased regardless of the initial conditions and the ratio of heat flow to compressed air leakage.
2) The patterns of air temperature change are determined by the difference between the amount of air leakage and the supplied/removed heat.

The decrease in the compressed air density over time was almost linear (Fig. 3a, 3b, 3c, 3e, 3f) or almost unchanged (Fig. 3d, 3g, 3 h). The linear character is explained by the value of the initial air pressure, which determines the power of its leakage and exceeds the supplied heat flux. At the same time, the absence of heat flux or its removal ($T_a \le T_0$) causes the invariance of the compressed air density relative to the environment.

Similar to the density, the decrease in compressed air pressure over time is polynomial for the initial pressure $p_0 \ge 200$ kPa (Fig. 3a, 3b, 3c, 3e, 3f) and almost linear for $p_0 < 200$ kPa (Fig. 3d, 3g, and 3 h). It also depends on the ratio of the air leakage flow to the supplied/removed heat flux, or its absence.

Regarding the temperature of the compressed air in the open thermodynamic system under consideration, its change in time has many differences, increasing with the increase in leakage time. When heat flow was supplied, depending on the initial pressure, the air temperature increased as follows:

- some decline during the initial period (Fig. 3a) or without it (Fig. 3b);
- A rapid increase in the initial period (Fig. 3c and 3b) with a gradual slowdown.

Without heat flow or under its removal, depending on the initial pressure, the air temperature increased with a rapid decrease over the initial period (Fig. 3e, 3f, 3g, 3 h) and then gradually increased.

As noted above, physical processes in an open thermodynamic system are generally possible under the following conditions (Fig. 2):

1) $G_{in} = G_{ex} + G_{out}$ and $dk/dT \neq 0$, where $G_{in} > 0$, $G_{ex} > 0$, and $G_{out} > 0$;
2) $dk/dT = 0$ and mass flow rates greater than 0;
3) $G_{in} = G_{out} = 0$, $G_{ex} > 0$ and dk/dT has any value.

Each process must be considered separately. In addition, the following options are possible:

- there is some leakage of gas medium (air) from the system, but the temperature of the environment compared to the temperature of the air in the system has not changed $T_a = T$;
- with the leakage of compressed air, it cools down due to heat exchange with the environment at $T_a < T$;
- some heating accompanies compressed air leakage due to its heat exchange with the environment at $T_a > T$.

Thus, while in the first variant, heat exchange with the environment does not affect the change in parameters in an open thermodynamic system, in other cases, it makes significant adjustments to the magnitude of changes in these parameters.

It can be noted that the time variation of the air pressure in the system may appear different with the same initial parameters (Table 1) than in Fig. 3, where $p_0 \ge 150$ kPa. Since the main factors determining the heat flow rate are the temperature difference T_a–T and the leakage rate determined by the pressure difference p_0–p_a, these values determine the nature of changes in the gas thermodynamic parameters in an open thermodynamic system. Fig. 4 shows the time variation of air pressure for the same conditions discussed above and for the conditions T_a–$T = 5$ and $p_0 \le 150$ kPa.

For these cases, Fig. 5 shows the patterns of air temperature change over time in an open system. As shown in the calculations, the dependence of the air density drop on time has a parabolic form, the shape of which coincides with the corresponding dependencies in Fig. 3.

For an open thermodynamic system at $p_0 \le 150$ kPa, the heat flow power significantly influenced the leakage power as the initial air pressure decreased. Initially, the pressure rises before declining, and the decrease is contingent on the initial pressure levels (Fig. 4). Concurrently, the air temperature in the system initially increased rapidly and then gradually stabilized to the ambient temperature. The growth rate was also influenced by the initial air pressure within the system.

The studies were conducted over a brief duration, assuming a constant ambient temperature. However, a prolonged analysis may result in variable ambient temperatures, necessitating the consideration of its impact on the pressure change of the gas medium within the thermodynamic system.

The analysis of time variation under ambient conditions of gas pressure, density, and temperature in an open thermodynamic system with leaks can be extended to industrial facilities with a higher pressure range, provided that the initial conditions are satisfied.

The gases have the following specific properties: compressibility, moisture (absolute and relative), viscosity (dynamic and absolute), heat capacity, and gas composition. These properties do not affect the adaptability of the model when appropriate adjustments are made. At the same time, the heat capacity of gas mixtures is calculated using the rule of additivity, which is inapplicable to the heat transfer coefficient (in the absence of reliable tabular data, it is determined experimentally).

The developed model can be generalized to other gases in an open

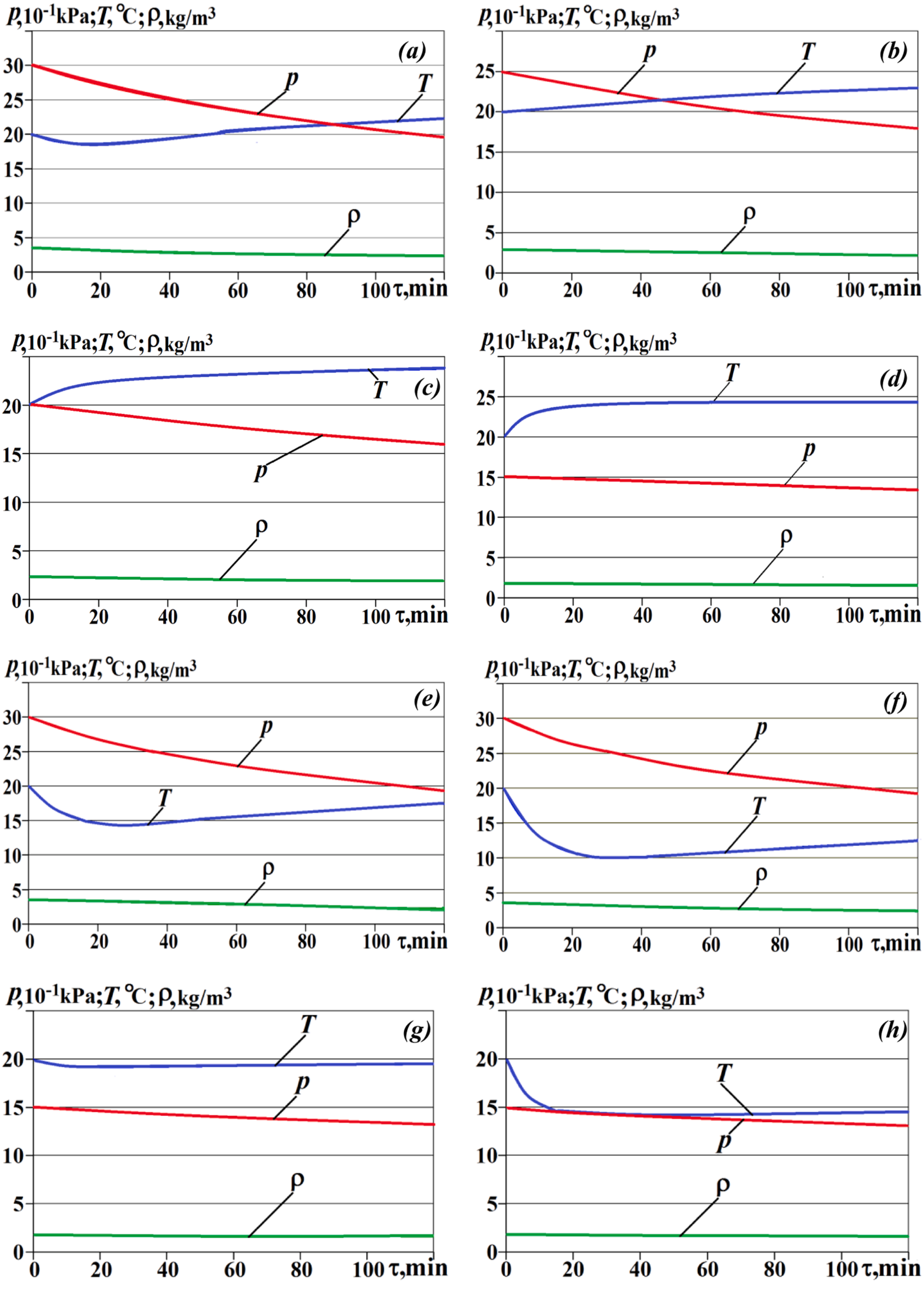

Fig. 3. Time variations of compressed air thermodynamic parameters in an open thermodynamic system at $p_0 = 300$ kPa and $T_a = 25$ °C (a); $p_0 = 250$ kPa and $T_a = 25$ °C (b); p_0=200 kPa and $T_a = 25$ °C (c); p_0 =150 kPa and $T_a = 25$ °C (d); $p_0 = 300$ kPa and $T_a = 20$ °C (e); $p_0 = 300$ kPa and T_a=15 °C (f); $p_0 = 150$ kPa and $T_a = 20$ °C (g); $p_0 = 150$ kPa and $T_a = 15$°C (h).

thermodynamic system in the presence of gas leakage in the laminar regime by introducing adjustments to the corresponding coefficients that consider the specific gas properties. For the treatment of transient environmental conditions with known functional dependencies on the temperature and pressure of the environment, this factor can be considered in the corresponding equations. From the point of view of the calculation method, this will not require any changes, but will significantly complicate the final analysis. In this case, the change in the thermodynamic parameters of the gas in the system due to leakage and heat exchange is influenced by the change in the time dependence of the

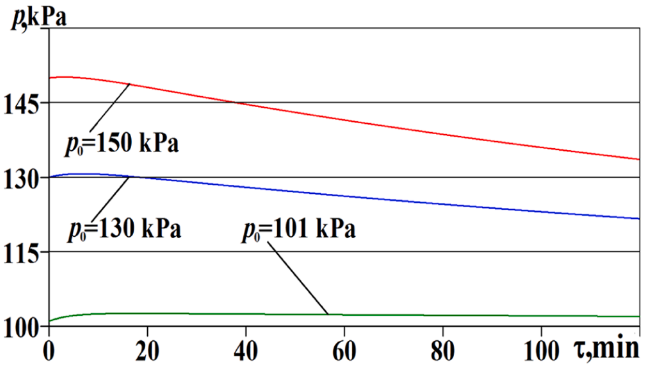

Fig. 4. Dependence of compressed air pressure in an open thermodynamic system at T_a=25°C for different initial pressures.

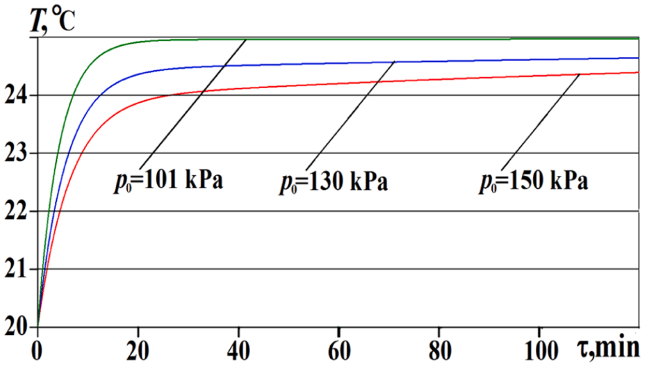

Fig. 5. Dependence of compressed air temperature in an open thermodynamic system at T_a=25 °C for different initial pressures.

environmental parameters.

Currently, there is no technology for real-time monitoring of forecasts obtained using the developed model. The development of this technology requires further research. There are no optimization methods for minimizing gas leaks based on this model, and their development is a task for further research. The mathematical model is based on the fundamental laws of thermo- and gas dynamics; it does not require machine learning to obtain the final results in leak detection and prediction.

4. Conclusion

A methodology for evaluating thermodynamic parameters in open systems with variable gas masses was designed to identify energy loss mitigation strategies. The novelty of the present study lies in the mathematical model obtained from differential equations of time variations of pressure, density, and temperature of gas in an open thermodynamic system, which determines the leakage rate of gaseous energy carriers depending on the parameters of the environment (pressure and temperature). The practical outcomes involve utilizing computational results from software in high-pressure system-monitoring devices. An example is a leakage detection device that uses a fixed-volume methodology, as discussed in reference studies [23,24]. The effectiveness of pneumatic energy use in technology largely depends on pressure differentials, which are influenced by air supply network leaks, thermal interactions, and energy consumer quantities. Thus, enhancing the evaluation methodologies for the impact of ambient temperature on the open-system parameters is essential. This improvement is vital for differentiating environmentally induced air fluctuations from those caused by leaks that affect pressure variations. The research outcomes address the critical challenges in the storage and distribution of gaseous energy carriers.

CRediT authorship contribution statement

Serhii M. Ponomarenko: Resources, Project administration, Data curation, Conceptualization. **Oleksandr V. Zhevzhyk:** Project administration, Methodology, Investigation, Funding acquisition, Formal analysis. **Iryna Yu Potapchuk:** Validation, Software, Methodology, Investigation. **Liudmyla B. Kabakova:** Supervision, Software, Resources. **Dmytro O. Yelatontsev:** Writing – review & editing, Visualization, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

The authors also express their gratitude to the staff of the tightness control laboratory of the Production Association Yuzhny Machine-Building plant named after A.M. Makarov (YUZHMASH) for their consultation and technical assistance in the studies.

Data availability

Data will be made available on request.

References

[1] Compressed air. Energy efficiency. Assessment, BSI British Standards, London, 2015. https://doi.org/10.3403/30182573u.
[2] An introduction to the compressed air energy storage, Computational Models for CO2 Geo-Sequestration & Compressed Air Energy Storage, CRC Press, 2014, pp. 463–474, https://doi.org/10.1201/b16790-23.
[3] K. Carpenter, Compressed Air Systems. Energy Management Handbook, River Publishers, Ninth edition. | Louisville, Kentucky : Fairmont Press, Inc., 2020, pp. 583–618, https://doi.org/10.1201/9781003151364-22 [2018].
[4] A.F. Bulat, O.I. Voloshyn, I.Y. Potapchuk, V.I. Yemelianenko, M.M. Zhovtonoha, O. V. Zhevzhyk, S. Manigandan, Mathematical modeling of the gas dynamic parameters of impinging heat-transfer medium jet in borehole thermal reaming process, Sci. Innov. 15 (2019) 17–23, https://doi.org/10.15407/scine15.03.017.
[5] A. Williams, Compressed Air Systems, in: Industrial Energy Systems Handbook, River Publishers, New York, 2023, pp. 357–406, https://doi.org/10.1201/9781003356431-15.
[6] M. Krisa, Taking a SMART approach to address compressed air leaks: measure the problem, then set out on a repair journey, Plant Eng 67 (2013) 43–50.
[7] D. du Plooy, P. Maré, J. Marais, M.J. Mathews, Local benchmarking in mines to locate inefficient compressed air usage, Sustain. Prod. Consum. 17 (2019) 126–135, https://doi.org/10.1016/j.spc.2018.09.010.
[8] A.F. Kuyuk, S.A. Ghoreishi-Madiseh, F.P. Hassani, A.P. Sasmito, L. Amiri, J. Templeton, Performance and economic assessment of large-scale deep-lake cooling systems: a Canadian example, Energy Procedia 158 (2019) 43–48, https://doi.org/10.1016/j.egypro.2019.01.033.
[9] N. Laurendeau, Summary of Classical thermodynamics, in: Statistical Thermodynamics, Cambridge University Press, Cambridge, 2012, pp. 409–414, https://doi.org/10.1017/cbo9780511815928.035.
[10] J.H. Vera, G. Wilczek-Vera, Basic concepts and definitions. Classical Thermodynamics of Fluid Systems, CRC Press, 2016, pp. 1–6, https://doi.org/10.1201/9781315399065-1.
[11] Y.N. Kaznessis, Elements of classical thermodynamics, in: Statistical Thermodynamics and Stochastic Kinetics, Cambridge University Press, Cambridge, 2011: pp. 309–311. https://doi.org/10.1017/cbo9781139015554.020.
[12] First law of thermodynamics, in: Thermodynamics, Cambridge University Press, 2020: pp. 254–316. https://doi.org/10.1017/9781316840979.007.
[13] E.B. Tadmor, R.E. Miller, R.S. Elliott, in Thermodynamics, Continuum Mechanics and Thermodynamics, Cambridge University Press, Cambridge, 2011, pp. 129–179, https://doi.org/10.1017/cbo9781139017657.007.
[14] S. Ponomarenko, O. Zhevzhyk, A. Vuginshteyn, O. Lutai, Thermodynamic analysis of the gas parameters distribution under leakage from a closed volume with rigid walls, E3S Web Conf. 109 (2019) 00071, https://doi.org/10.1051/e3sconf/201910900071.
[15] O.V. Zamytskyi, B.M. Litovko, M.Y. Lider, M.I. Shepelenko, Mathematical modeling of the process of compressed air flowing through the pipeline as an element of the pneumatic network, Nauk. Visnyk Natsionalnoho Hirnychoho Universytetu 3 (2019) 54–60, https://doi.org/10.29202/nvngu/2019-3/8.

[16] Q. Hou, D. Yang, X. Li, G. Xiao, S.C.M. Ho, Modified leakage rate calculation models of natural gas pipelines, Math. Probl. Eng. 2020 (2020) 1–10, https://doi.org/10.1155/2020/6673107.

[17] J.C. de Almeida, A.P. Sardá, H.S.F. de Oliveira, Volume of natural gas released into the atmosphere in leakage situations - review and analysis, Int. J. Adv. Eng. Res. Sci. 9 (2022) 173–177, https://doi.org/10.22161/ijaers.911.22.

[18] G.I. Usiabulu, J. Ogbonna, V. Aimikhe, E. Okafor, L. Nosike, Estimation of gas leak volume to quantify gaseous fluid flow in processing plants, J. Appl. Sci. Environ. Manage. 27 (2023) 1761–1769, https://doi.org/10.4314/jasem.v27i8.21.

[19] S. Wang, Y. Li, C. Liu, Z. Cui, Experimental research on leakage and diffusion characteristics and leakage quantity prediction of underwater gas pipeline, J. Appl. Sci. Eng. 23 (2020) 143–152, https://doi.org/10.6180/jase.202003_23(1).0015.

[20] H. Rottländer, W. Umrath, G. Voss, Fundamentals of Leak Detection, Leybold GmbH, 2016.

[21] K. Fojtášek, L. Dvořák, L. Krabica, Comparison and mathematical modelling of leakage tests, EPJ Web Conf 213 (2019) 02019, https://doi.org/10.1051/epjconf/201921302019.

[22] B. Gu, X. Huang, Investigation of leak detection method by means of measuring the pressure increment in vacuum, Vacuum 80 (2006) 996–1002, https://doi.org/10.1016/j.vacuum.2006.01.005.

[23] A. Bulat, O. Voloshyn, S. Ponomarenko, D. Gubenko, New-generation technique and technology for leakage tests. Mining of Mineral Deposits, CRC Press, 2013, pp. 1–4, https://doi.org/10.1201/b16354-1.

[24] S. Ponomarenko, I. Potapchuk, O. Zhevzhyk, L. Kabakova, A. Potapchuk, Influence of the leakage in air supply networks on the efficiency of application of pneumatic backfill equipment, Min. Min. Deposits 17 (2023) 83–92, https://doi.org/10.33271/mining17.04.083.