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[2 ó 4, 10 ó 11],

$$(\quad),$$

$$(\quad).$$

$$\cdot$$

$$\vdots$$

$$C = \sum_{i,j \in V} \sum_{\omega \in W_{ij}} X_{ij\omega} \cdot l(\omega) \rightarrow \min, \quad (1)$$

$$l(\omega) \text{ ó } \omega;$$

$$\sum_{\omega \in W_{ij}} X_{ij\omega} \quad G(V, E).$$

$$\cdot$$

$$G(V, E) \text{ ó } V$$

$$E. \quad R(e).$$

$$G(V, E) \quad P_{ij}, \quad i, j \in V.$$

$$W_{ij} \text{ ó } i \neq j, \quad \omega \text{ ó }$$

$$W_{ij}.$$

$$X_{ij\omega} \quad P_{ij},$$

$$\omega,$$

$$\sum_{\omega \in W_{ij}} X_{ij\omega} = P_{ij}. \quad (2)$$

$$e \quad \omega,$$

$$I_\omega(e) = \begin{cases} 1, & e \in \omega; \\ 0, & e \notin \omega, \end{cases} \quad (3)$$

$$e \quad W_{ij}$$

$$\sum_{\omega \in W_{ij}} X_{ij\omega} \cdot I_\omega(e), \quad (4)$$

e

$$N(e) = \sum_{i,j \in V} \sum_{\omega \in W_{ij}} X_{ij\omega} \cdot I_\omega(e); \quad e \in E. \quad (5)$$

$$\bar{N}(e) \leq N(e), \quad e \in E.$$

:

$$N(e) \leq \bar{N}(e), \quad e \in E. \quad (6)$$

$$, \quad l(\omega) \leq \omega, \quad l(\omega) = \sum_{e \in \omega} R(e),$$

$$Pr = \sum_{i,j \in V} \sum_{\omega \in W_{ij}} X_{ij\omega} \cdot l(\omega) \quad (7)$$

$$P_{ij},$$

$$i, j \in V \quad G(V, E).$$

$$E_* \leq ,$$

,

$$L(E_*) = \sum_{e \in E_*} R(e) \quad (8)$$

$$, \quad X_{ij\omega},$$

$$Pr(E_*) - L(E_*) \quad , \quad (2)$$

(6).

$$\begin{pmatrix} L(E_*) \\ Pr(E_*) \end{pmatrix} \rightarrow \min, \quad (9)$$

(2) (6).

$$,\sum_{i,j\in V}i < j\,,$$

$$(\quad), \qquad i>j\,,$$

$$(\quad).$$

$$G(V,E) \qquad \qquad \qquad , \qquad \qquad \qquad \mathfrak{o}$$

$$,\qquad \qquad R(e)\leq \mathfrak{o}\qquad \qquad ,\qquad P_{ij}$$

$$i\qquad j\qquad e\,,\qquad L(E_*)$$

$$,\quad Pr(E_*)$$

$$[12\leq 14].$$

$$(i\rightarrow j)$$

$$\vdots$$

$$\min_{e\in E\setminus E_*}\frac{Pr(E_*\cup\{e\})-Pr(E_*)}{l(E_*\cup\{e\})-l(E_*)}. \tag{10}$$

$$.\,1.$$

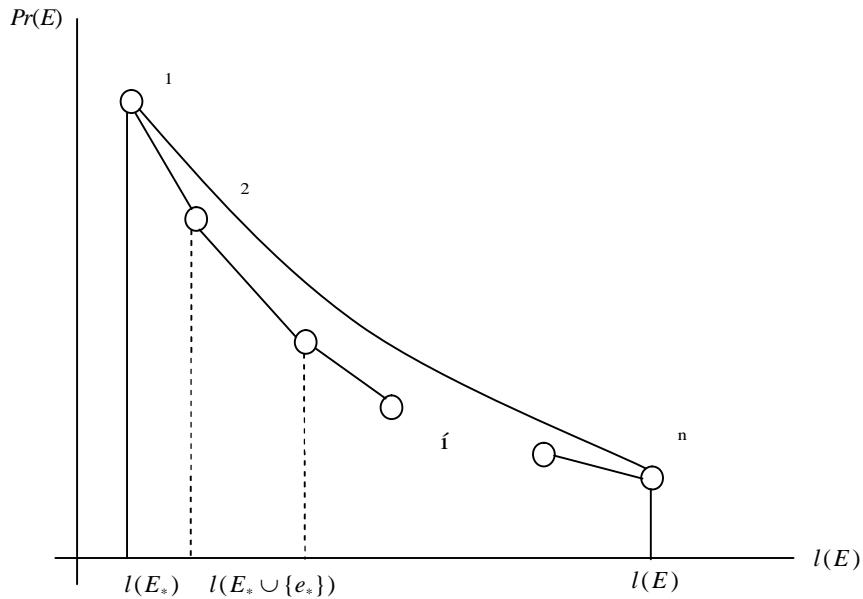
$$e_*\qquad \qquad \qquad ,\qquad \qquad E_* = E_* \cup \{e_*\},$$

$$A_2\,(\ldots .1)\ldots .\qquad \qquad |E\setminus E_*|+1.$$

$$,\qquad Pr(E_*\cup\{e\})\leq Pr(E_*)\qquad \qquad L(E_*\cup\{e\})\leq L(E_*)\,.$$

$$i\qquad j$$

$$(i\rightarrow j)\qquad N(N+1)/2\,,\qquad N=|V|.$$



. 1 ó i
 j

,

. N ó $G(V, E)$; zn ó

; zk ó

$zn - zk$.

1) KP ó ; KW ó

2) $zn - zk$.

$KW = \{ \{zn\} \}$.

3) $w \in KW$

KW .

4) z_1, z_2, \dots, z_p w .

5) $z_i, i = \overline{1, p}, p < N$:

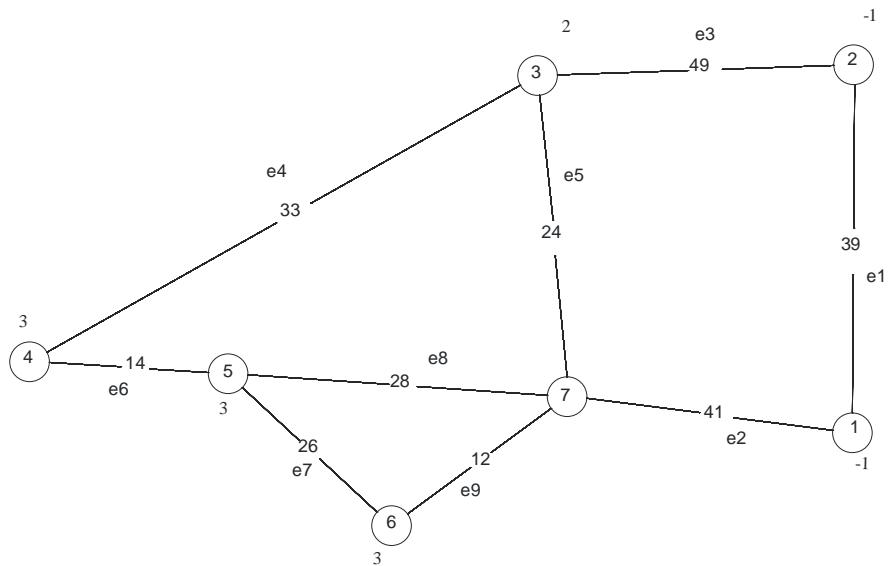
5.1) $z_i = zk, KP \cup \{w \cup z_i\}$ ó

;

5.2) $z_i \cap w = \emptyset, KW \cup \{w \cup z_i\}$ ó

6) $KW = \emptyset$ ó ,
 , 5.2.

, , . 2.



. 2 ó

G , $N = 7$ (. 2).

$zn = 2$ $zk = 4$.

$KW = \{\{1\}\}$. $w = \{1\};$

$KW = \{\}$.

$w = \{1\}$ $z_1 = 3$, $z_2 = 1$;

$p = 2$.

$z_1 \neq zk$, $z_2 \neq zk$, $KP = \{\}$.

$KW = \{\{2,3\}, \{2,1\}\}$.

$w = \{2,3\};$

$$KW = \{\{2,1\}\}.$$

$$w = \{2,3\} \quad z_1 = 4, \quad z_2 = 7;$$

$$p = 2.$$

$$z_1 = zk, \quad z_2 \neq zk, \quad KP = \{2,3,4\}.$$

$$KW = \{\{2,3,7\}, \{2,1\}\}.$$

$$w = \{2,3,7\} \quad - \quad KP = \{2,3,4\},$$

$$KW = \{\{2,3,7,5\}, \{2,3,7,6\}, \{2,1\}\}.$$

$$w = \{2,3,7,5\} \text{ ó } KP = \{\{2,3,4\}, \{2,3,7,5,4\}\}.$$

$$w = \{2,3,7,6\} \text{ ó } KW = \{\{2,3,7,6,5\}, \{2,1\}\}.$$

$$w = \{2,3,7,6,5\} \quad \text{ó}$$

$$KP = \{\{2,3,4\}, \{2,3,7,5,4\}, \{2,3,7,6,5,4\}\},$$

$$KW = \{\{2,1\}\}.$$

$$KW = \{\{2,1\}\} \quad z_1 = 7,$$

$$KW = \{\{2,1,7\}\}. \quad KW = \{\{2,1,7\}\}$$

$$z_1 = 3, \quad z_2 = 5, \quad z_1 = 6. \quad KW = \{\{2,1,7,3\}, \{2,1,7,5\}, \{2,1,7,6\}\}.$$

$$z_1 \neq zk, \quad z_2 \neq zk, \quad z_3 \neq zk,$$

$$KP = \{\{2,3,4\}, \{2,3,7,5,4\}, \{2,3,7,6,5,4\}\}.$$

$$w = \{2,1,7,3\},$$

$$z_1 = 4, \quad KP = \{\{2,3,4\}, \{2,3,7,5,4\}, \{2,3,7,6,5,4\}, \{2,1,7,3,4\}\},$$

$$KW = \{\{2,1,7,5\}, \{2,1,7,6\}\}.$$

$$w = \{2,1,7,5\},$$

$$z_1 = 4,$$

$$KP = \{\{2,3,4\}, \{2,3,7,5,4\}, \{2,3,7,6,5,4\}, \{2,1,7,3,4\}, \{2,1,7,5,4\}\},$$

$$KW = \{\{2,1,7,6\}\}.$$

$$w = \{2,1,7,6\},$$

$$z_1 = 5, \quad KW = \{ \{2,1,7,6,5\} \}. \quad z_1 \neq zk,$$

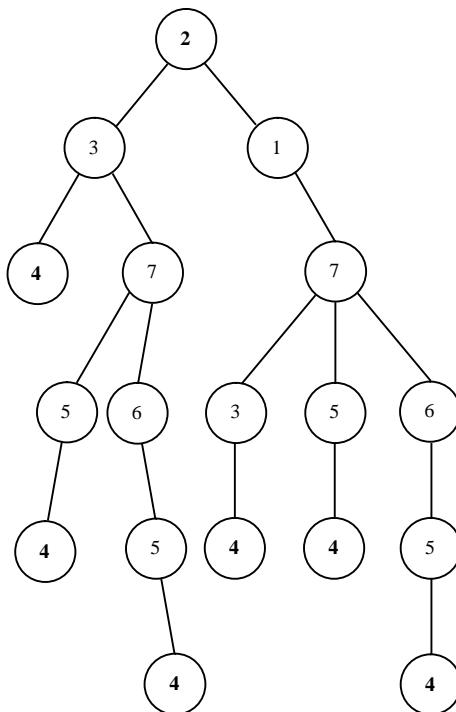
$$\cdot \quad KW = \{ \{2,1,7,6,5\} \}$$

$$z_1 = 4, \quad KW = \{ \ }, \quad w = \{2,1,7,6,5,4\}$$

,

$$KP = \{ \{2,3,4\}, \{2,3,7,5,4\}, \{2,3,7,6,5,4\}, \{2,1,7,3,4\}, \{2,1,7,5,4\}, \{2,1,7,6,5,4\} \}.$$

. 3



. 3 6

, 2 4 6

.

$$H(V, E_*) \quad W(E_*) \quad , \quad \omega \quad \delta$$

$$W(E_*), \quad , \quad e \in \omega,$$

$$X(e, \omega).$$

$$N(e) = \sum_{\substack{\omega \in W(E_*) \\ e \cap \omega = e}} X(e, \omega). \quad (11)$$

$$N(E_*) = \max_{e \in E_*} N(e),$$

$$N(E_*) \leq \bar{N}, \quad H(V, E_*)$$

$$\cdot, \quad \quad \quad \bar{N}$$

$$, \quad \quad \quad H(V, E_*)$$

$$N(e) \leq \bar{N}(e), \quad \forall e \in E_*.$$

$$N(e, \omega)$$

$$X(e, \omega) = \sum_{\rho=1}^{j(e)} \sum_{\nu=j(e)+1}^{m_\omega} P(i_\rho, i_\nu), \quad (12)$$

$$P(i_\rho, i_\nu) \text{ ó } i_\rho \quad i_\nu;$$

$$(j(e), j(e)+1) \text{ ó } \omega, \quad e \in \omega;$$

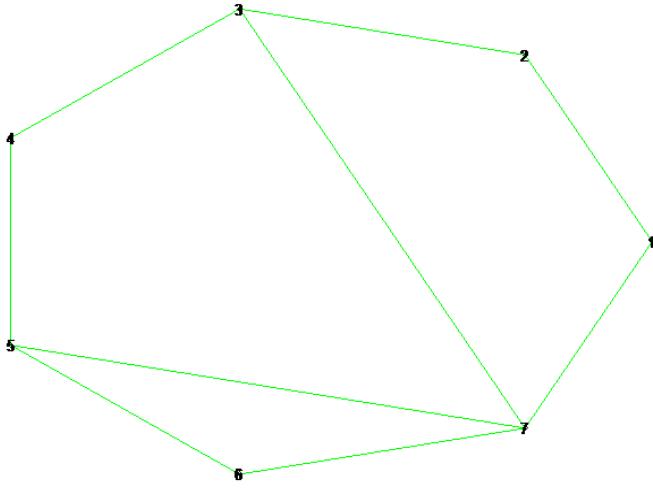
$$m_\omega \text{ ó } \omega.$$

$$G \text{ (} \dots 4,$$

$$7$$

$$\vdots$$

$$e1=\{1,2\}, \quad e2=\{1,7\}, \quad e3=\{2,3\}, \quad e4=\{3,4\}, \quad e5=\{3,7\}, \quad e6=\{4,5\}, \quad e7=\{5,6\}, \\ e8=\{5,7\}, \quad e9=\{6,7\}.$$



$$\text{. 4 ó }$$

$$R(e) = \begin{bmatrix} 0 & 39 & 0 & 0 & 0 & 0 & 41 \\ 39 & 0 & 49 & 0 & 0 & 0 & 0 \\ 0 & 49 & 0 & 33 & 0 & 0 & 24 \\ 0 & 0 & 33 & 0 & 14 & 0 & 0 \\ 0 & 0 & 0 & 14 & 0 & 26 & 28 \\ 0 & 0 & 0 & 0 & 26 & 0 & 12 \\ 41 & 0 & 24 & 0 & 28 & 12 & 0 \end{bmatrix} \quad P_{ij} = \begin{bmatrix} 0 & 2 & 20 & 0 & 5 & 5 & 14 \\ 1 & 0 & 0 & 18 & 40 & 0 & 17 \\ 0 & 0 & 0 & 11 & 7 & 3 & 10 \\ 2 & 11 & 14 & 0 & 5 & 7 & 16 \\ 3 & 10 & 11 & 5 & 0 & 20 & 8 \\ 0 & 21 & 8 & 23 & 0 & 0 & 0 \\ 6 & 7 & 0 & 0 & 8 & 10 & 0 \end{bmatrix} \quad : \quad$$

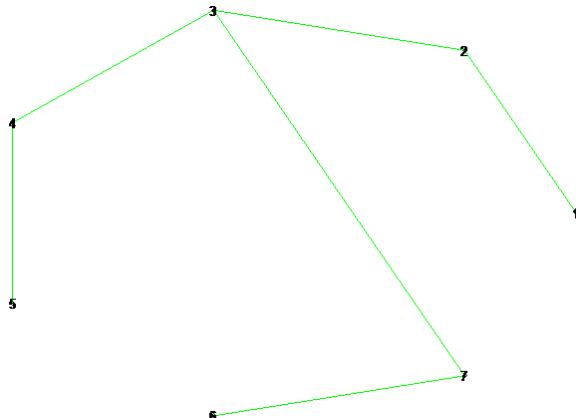
$$\bar{N}(e) = 140.$$

Maple [15 ó 17],

$$[2 \text{ ó } 3] \quad H(V, E_*), \quad .5.$$

$$H(V, E_*) \quad L(E_*) = 171,$$

$$E_* = \{e1, e3, e4, e5, e6, e9\}.$$



. 5 6

$$i \quad j, \quad i < j,$$

$$(\quad), \quad H(V, E_*)$$

. 1.

. 1 -

\	2	3	4	5	6	7
1	[e1]	[e1, e3]	[e1, e3, e4]	[e1, e3, e4, e6]	[e1, e3, e5, e9]	[e1, e3, e5]
2	ó	[e3]	[e3, e4]	[e3, e4, e6]	[e3, e5, e9]	[e3, e5]
3	ó	ó	[e4]	[e4, e6]	[e5, e9]	[e5]
4	ó	ó	ó	[e6]	[e4, e5, e9]	[e4, e5]
5	ó	ó	ó	ó	[e6, e4, e5, e9]	[e6, e4, e5]
6	ó	ó	ó	ó	ó	[e9]

:	1	2	[e1]
2;	1	3	[e1, e3]
20;	1	5	[e1, e3, e4, e6]
5;	1	6	[e1, e3, e5,
e9]	5;	7	[e1, e3,
e5]	14;	4	[e3, e4]
18;	2	5	[e3, e4, e6]
40;	2	7	[e3, e5]
17;	3	4	[e4]

11;	3	5	[e4, e6]	
7;	3	6	[e5, e9]	
3;	3	7	[e5]	
10;	4	5	[e6]	
5;	4	6	[e4, e5, e9]	
7;	4	7	[e4, e5]	
16;	5	6	[e6, e4, e5,	
e9]	20;	5	7	[e6, e4,
e5]	8.			

. 2.

. 2 -

	e1	3	4	5	6	9
	39	49	33	24	14	12
	46	119	132	95	65	35

$Pr(E_*)=15591$

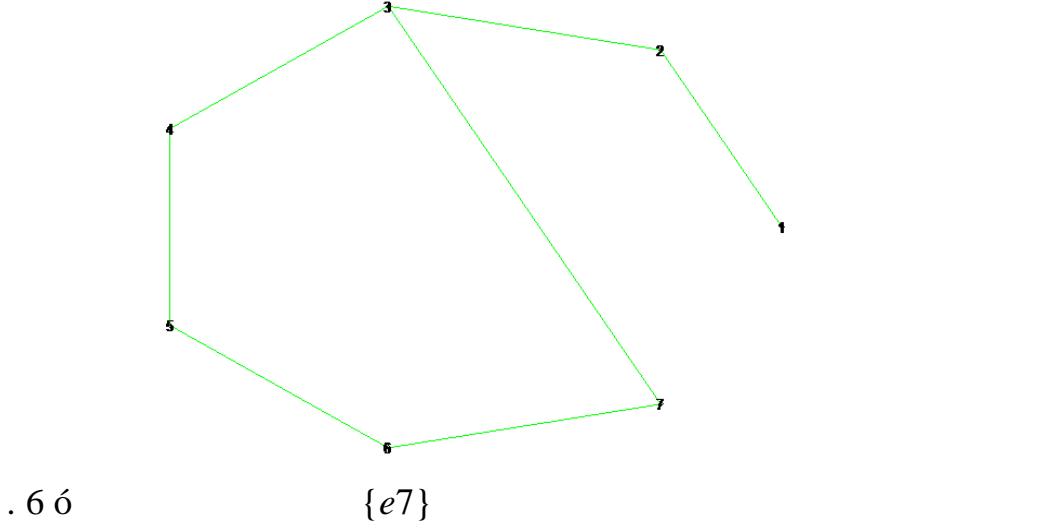
, $H(V, E_*)$ {e2, e7, e8}.

:

$QE=\{\{e2\}, \{e7\}, \{e8\}, \{e2, e7\}, \{e2, e8\}, \{e7, e8\}, \{e2, e7, e8\}\}.$

, $H(V, E_*)$

$$H(V, E_*) \quad \{e7\}, \quad 26, \quad . 6.$$



$$E_* = \{e1, e3, e4, e5, e6, e7, e9\}.$$

:	1	2	$[e1]$
2;	1	3	$[e1, e3]$
20;	1	5	$[e1, e3, e4, e6]$
5;	1	6	$[e1, e3, e5,$
$e9]$	5;	1	$[e1, e3,$
$e5]$	14;	2	$[e3, e4]$
18;	2	5	$[e3, e4, e6]$
40;	2	7	$[e3, e5]$
17;	3	4	$[e4]$
11;	3	5	$[e4, e6]$
7;	3	6	$[e5, e9]$
3;	3	7	$[e5]$
10;	4	5	$[e6]$
5;	4	6	$[e6, e7]$

7; 4 7 [e6, e7, e9]
 16; 5 6 [e7]
 20; 5 7 [e7, e9]
 8.

. 3.

. 3 -

	e 1	3	e 4	5	6	7	9
	3 9	49	3 3	2 4	1 4	2 6	1 2
	4 6	11 9	8 1	4 9	8 0	5 1	3 2

$$Pr(E_*) = 14304$$

- .

$$, \quad H(V, E_*)$$

QE.

. 4.

8 (

G)

$$[12]. \quad 1 \quad H(V, E_*),$$

$$L(E_*) = 171 \quad ,$$

$$Pr(E_*) = 15591 \quad - ; \quad 2 \quad H(V, E_*) \quad (\quad H(V, E_*)$$

{e7}),

$$L(E_*) = 197 \quad ,$$

$$Pr(E_*) = 14304 \quad - \quad .$$

. 4 -

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	,	,	,	,
	$H(V, E_*)$			
1	-	-	171	15591
2	7	26	197	14304
3	8	28	199	14442
4	e2	41	212	14067
5	e7, e8	54	225	15364
6	e2, e7	67	238	12215
7	e2, e8	69	240	12303
8	e2, e7, e8	95	266	11925

$$6 \quad . 4 (\quad H(V, E_*) \quad \{e2, e7\});$$

$$L(E_*) = 238 \quad , \quad Pr(E_*) = 12215 \quad - \quad ;$$

$$5 \quad . 4 (\quad H(V, E_*) \quad \{e2, e7, e8\}),$$

$$L(E_*) = 266 \quad , \quad Pr(E_*) = 11925 \quad - \quad ;$$

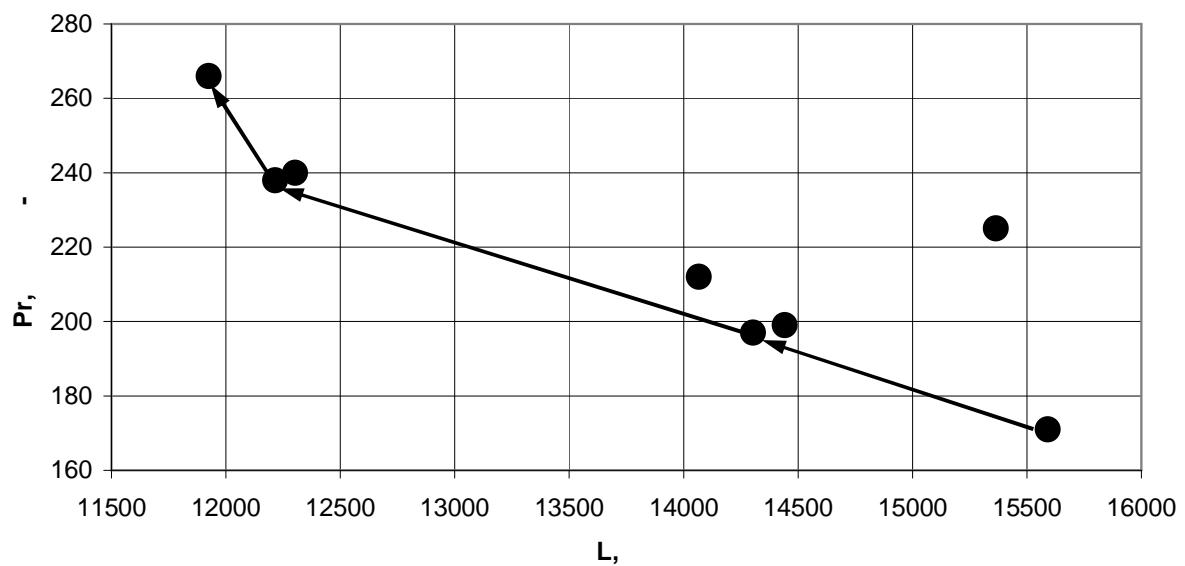
$$8 \quad . 4 (\quad H(V, E_*) \quad \{e2, e4, e9\});$$

$$L(E_*) = 266 \quad , \quad Pr(E_*) = 11925 \quad - \quad ,$$

« »,

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. 7.



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 (6) 1, 2, 3, 5, 8 . 4,

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(9) (2) (6)
1, 2, 3, 5, 8 . 4,

$G(V, E)$

P_{ij} ó , $i \neq j$, $L(E_*)$
,

$Pr(E_*)$

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2. , . . . : . /
. . // ó .: , 1979. ó 143 .
3. . . . / . . . //
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ó . 31-34.
- 5.** . . . / . . // ó
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- 6.** , . . [] /
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- 7.** , . . [] / . . , . . //
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. - , 2010. ó 180-188
- 8.** , . . / . . , . // ó
. , 1984. ó 392 .
- 9.** , . . /
. // ó . , 1978. ó 432 .
- 10.** , . . / . . //
4, 2009 ó . 112 ó 116.
- 11.** , . . / . . // " "
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- 12.** , . . « -
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- , 1982 . ó 256 .
- 13.** , . . « -
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∅ , 2004. ó 22 .

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UDC 665.9

RATIONAL TRAINFLOW FORMATION ON THE RAILWAY NETWORK

Yu. V. Chibisov

Mathematical model of the rational trainflow distribution on the rail network was offered in this article. The main purpose of research work is development of the algorithm which will help to make the shortest paths of the trainflows, which will provide a minimum expenditure of energy resources to the promotion of trains. The trainflows distribution on the network was made with the help of the graph theory. The minimum of the train-kilometers was chosen as the optimality criterion. Im.:7 : Bibliogr.: 617.

Keywords: railway network, train flow, mathematical model, rational distribution, vector optimization.