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CONSTRUCTIVE-SYNTHESIZING REPRESENTATION OF GEOMETRIC FRACTALS

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Abstract. *A constructive-production approach, which is more general than other well-known approaches, is proposed to generate fractals. Possibilities are shown for using a large variability of attributes and initial elements of formation of fractals, as well as combining fractals into multifractals. The possibilities of generating fractals are extended by eliminating the constraints necessary in other approaches. The proposed approach allowed to establish several previously unknown properties of fractional dimension that consist of the possibility of changing it in the process of generation of a fractal and a mismatch of fractional dimensions of the form limit during generation and the limiting fractal. A simple definition of a deterministic geometric fractal is given. This definition takes into account all the properties characterizing such a fractal.*

Keywords: *constructor, fractal, multifractal, fractional dimension, Sierpinski triangle, fractal geometry.*

INTRODUCTION

The rapid development of fractal geometry was initiated by B. Mandelbrot [1]. Fractals are found out in many natural objects and processes. Fractal models are used in many scientific areas such as biology, architecture, medicine, materials science, nuclear physics, astronomy, mathematics, information systems, etc.

There are several alternative approaches to the formation of fractals, namely, algorithmic [1–5]; functional algorithmic with the use of a system of iterated functions based on a collection of contracting mappings [2, 4, 5]; L-systems [2]; contracting affine automata [6].

The algorithmic approach was formed when describing first classic fractals such as the Sierpinski triangle and carpet, Koch snowflake, etc. Algorithms of formation of these fractals were proposed.

The functional-algorithmic approach consists of representing fractal geometry using classical mathematics.

A rather new direction of formation of fractals, namely, contracting affine automata, allows to form fractals based on affine transformations and has its theoretical basis in the form of finite automata.

L-systems proposed by A. Lindemayer [7] as an essential modification of formal grammars received the greatest practical application in computer graphics.

In [8–10], the foundations of constructive-synthesizing modeling (CSM) were laid within the framework of which it is possible to model any constructions and constructive processes in the fields of information technologies, building, mechanical engineering, robotics, biology, etc. The proposed apparatus allows to formalize processes and results of formation of constructions of various nature by connecting elements of constructions and taking into account properties of elements, their aggregates (forms), and connections.

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This article considers the application of CSM to the formation of fractals. This approach is closest to L-systems. It is more adapted for work with attributes and is more flexible in specifying the process of inference.

CONCEPT OF A CONSTRUCTOR

We call the following triple a constructor [8]:

$$C = \langle M, \Sigma, \Lambda \rangle,$$

where M is a heterogeneous replenishable carrier, Σ is the signature of operations (and the corresponding relations) of connection, substitution, and inference and also operations over attributes, Λ is a set of (formal and informal) statements of construction dataware (CDW). CDW includes an ontology, an objective, rules, constraints, initial conditions, and conditions of completion of construction. The ontology of the generalized constructor Λ is presented in [8]. In this case, a number of refining transformations of the generalized constructor are provided.

Specialization determines the following ontology of a subject area: the semantic nature of its carrier, the objective common to its family of problems, a finite set of operations and their semantics, attributes of operations, the order of their execution, and constraints. The specialization operation ${}_S\mapsto$ is executed by an external executor.

Interpretation lies in linking signature operations with algorithms that execute some algorithmic structure [10]. During interpretation, the models of the constructor and internal executor of the process of construction are linked. The result of an interpretation ${}_I\mapsto$ is a constructive system. The interpretation operation is executed by an external executor.

A concretization of a constructor provides for specifying concrete rules, constraints, initial conditions, and also the conditions of construction completion and concretization of the following carrier element base: the sets of its nonterminal and terminal symbols with their attributes and (if necessary) the values of attributes. After interpreting and concretizing ${}_K\mapsto$, which are performed by an external executor, the constructive system has everything necessary for independent formation of constructions.

The implementation ${}_R\mapsto$ performed by an internal executor of the system consists of the formation of a construction from carrier elements by executing algorithms related to signature operations. Only a previously specialized, interpreted, and concretized constructor can be implemented.

SPECIALIZATION OF CONSTRUCTORS OF GEOMETRIC OBJECTS IN THE SPACE $\mathbf{R}^2$

The specialization of a constructor lies in the formation of geometric figures (constructions) in the space $\mathbf{R}^2$ including the following fractals:

$$C = \langle M, \Sigma, \Lambda \rangle \ {}_S\mapsto C_G = \langle M_G, \Sigma_G, \Lambda_G \rangle,$$

where $\Lambda_G = \Lambda \cup \Lambda_1 \cup \Lambda_2$, $\Lambda_1 = \{M_G \supset \mathbf{R}^2 \cup T_G \cup F_G \cup K_G$; $\Sigma_G = \{\Xi_G, \Theta_G, \Phi_G, \{\rightarrow, \lrcorner\}\}$, $\Phi_G = \{\bullet, /, +, -, \circ\}$, and $\Theta_G = \{\Rightarrow, |\Rightarrow, ||\Rightarrow\}\}$. Here, we have the following sets: T_G are conditionally indivisible graphic construction elements; F_G and K_G are intermediate forms and constructions, respectively; Ξ_G are relations of relative positions of geometric figures (binding of elements) that are specified graphically; Θ_G are the operations of substitution and inference, Φ_G are operations over attributes, and also the relations of substitution ($\rightarrow$) and attributiveness ($\lrcorner$).

The CDW component Λ_2 contains definitions, additions, and constraints that refine the alphabet, carrier attributes, and substitution relations and determine the peculiarity of executing the substitution and inference operations.

A figure or an object is understood to be a compact connected set in $\mathbf{R}^2$.

The presence of an attribute w at a carrier element m is denoted by ${}_w m$ (the identifier m with the attribute w), and the designation $w \lrcorner m$ testifies that w is an attribute of the identifier m.

The terminal alphabet T_G consists of a set of geometric objects with attributes.

Moreover, the possibilities of formation of fractals are extended by eliminating the constraints required for other approaches, namely,

— the initial elements of formation of fractals must be represented by essentially disjoint sets;

— only contracting mappings are used.

The proposed approach has allowed to establish several previously unknown properties of fractional dimension, namely,

— the possibility of changing it during the generation of the limiting fractal and convergence, in particular, to 1;

— the noncoincidence of fractional dimensions of the limit of forms during generation and the limiting fractal $\lim_{N \to \infty} (d(f_N)) \neq d(\lim_{N \to \infty} (f_N))$.

Using CSM has allowed to give a simple definition of a deterministic geometric fractal taking into account all the properties characterizing it.

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