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Fatigue lifetime model under a complex loading with application of the amalgamating safety indices rule

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Abstract

The problem of multiaxial fatigue is considered from the standpoint of amalgamating indicators of system reliability. The lifetime model is derived from the rule of amalgamating resource indices of safety. The load is represented as a combination of individual subprocesses of simple types of deformation with their amplitudes and stress ratio. A model of lifetime in multiaxial fatigue has been developed, which takes into account the parameters of the deformation cycle shape, the type of process (in-phase, disproportionate, constant static stress). The possibility is confirmed for obtaining the parameters of the model of multiaxial fatigue when tested for three-point bending under conditions of variation of the multiplicity of span. According to this scheme, fatigue tests were performed for prismatic steel samples 09G2 and 40H. The fatigue resistance parameters were found for them, and also there were found the ratio of the fatigue limit for tangential stresses and the fatigue limit for normal bending stresses, which is equal to 0.385.

Keywords: multiaxial fatigue, safety index, shear stress, bending

1. Introduction

The problem of this work dedicated, arose in connection with the transition from the methods of classical reliability to the methods of structural reliability, when it is determined not by testing the system as a whole, but by

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the indicators of individual elements of the system. It is necessary to amalgamate (merge) individual indicators into a single one, as it is more convenient to control the reliability of the whole system during maintenance. This is done according to certain algorithms and rules, which are a specific section of the theory of reliability and risk. From the methodological point of view, the task of amalgamating the reliability indicators of individual elements is equivalent to the task of amalgamating the actions of individual components of a set of degradation processes.

Most of the really functioning elements of structures and parts of machines suffer from a complex cyclic loading and a combination of degradation processes. It should be understood that the operating conditions are much more complicated than the design schemes that are used in the design to confirm the operability. Usually, the calculation is made to prevent failures from the main (dominant) degradation process or type of loading. However, effective technical diagnostics requires consideration of all factors influencing the operation process. In this situation, it is necessary to combine individual indicators of damage processes into a single solution, into a single comprehensive indicator for determination the technical condition of the mechanical system.

Lot of constructive elements of the rolling equipment function in the conditions of a difficult stress state. In particular, these are basic structures, such as the stands of working stands. These are the drive units of the rolls - connecting spindles, shafts, couplings. We can also add rolling tools - rolls. All of these objects suffer from cyclical normal and exact tensions. At the stage of defect growing, such conditions provoke the destruction of the mixed type, when I, II and III modes are observed at the same time. For mixed destruction, the authors developed the so-called method of "pure modes", which was used to assess the survivability of rolling mills, as well as the frames of the stands of the pipe rolling machine (Belodedenko et al. (2018), Rakhmanov et al. (2020)). The method consists in first determining the survivability curves for individual pure modes, after which the number of growth cycles to the critical crack size is determined by a certain combination algorithm. This idea is proposed to be implemented in the algorithm for determining the number of load cycles before the crack appearance.

When solving the problem of complexes of deformations and damages, the operation of amalgamating ($\bigcup$) can happen at different levels or stages (Fig. 1). Thus, to ensure strength, it is sufficient to combine, according to certain hypotheses, the indicators of the stress-strain state (σ_i, τ_i, ε_i, γ_i) or margins of safety (n_σ, n_τ). In some cases, we can use the parameters of *DP* damage, the calculation of which has its own specifics. This gives the strength of the system (n_Σ) or equivalent stresses (σ_{eq}, τ_{eq}, ε_{eq}, γ_{eq}). To ensure the durability of the system, the damage D_i of individual degradation processes is combined (summed up). For processes of different origin, the combination of individual reliability indicators R_i is used, after which the reliability of the system R_Σ is determined. This principle has also been used to determine the probability of failure in the complex stress state (CSS) (Sosnovsky (1987).

The last indicator has a physical meaning as the relative number of failures of the same type of objects. For unique objects, such content is lost. Therefore, it is necessary to move to risk analysis methods. Given the severity of the failures, the risk can be assessed ρ_Σ or safety β_Σ systems on separate indicators ρ_i and β_i.

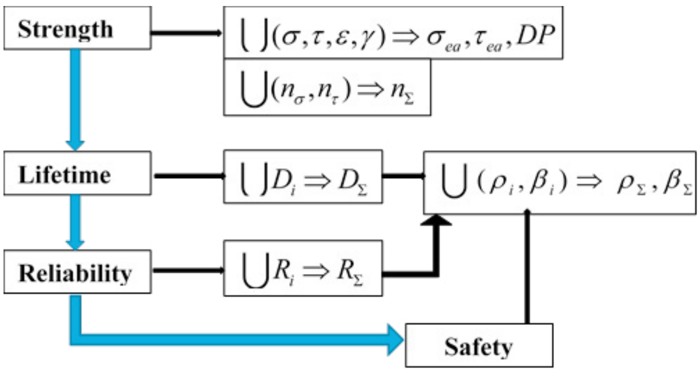

Fig. 1. Scheme for solving the problem of complex loading and damage by amalgamating ($\bigcup$) indicators of properties of mechanical systems.

The last indicator is called the resource safety index (RSI) rule. Its concept was developed by the authors and was used to summarize fatigue damages under non-stationary loading (Belodedenko et al. (2019)). In this case, the load is represented as a combination of individual subprocesses with their amplitudes and stress ratio. This approach has also been used to predict mixed fracture survivability.

The work was aimed at spreading the rule of amalgamating resource indices of safety in case of multiaxial fatigue. It is proposed to test this algorithm during fatigue tests according to the three-point bending scheme (3 pointbending, 3PB) with a variation of the multiplicity of the span γ_l, which provides the variability of the ratio of normal σ and tangential stresses τ.

2. The rule of amalgamating recourse safety indices

Proactive maintenance strategies use a complex indicator to assess the technical condition RSI β_{Ri}, calculated for the reliability R and for the degradation process i (Belodedenko et al. (2019)). For their complex (system) the general safety index will be:

$$\beta_{\Sigma R} = \lg N_{\Sigma R} = \lg\left(\sum \frac{U_i}{10^{\beta_{Ri}}}\right)^{-1}, \tag{1}$$

where U_i is the relative importance of failure (criticality) with probability Q_i=1-R_i.

For cyclic processes, the criticality U_i is determined by the relative duration of the process c_i, which corresponds to the level of the block load. Also, the criticality depends on the accumulated damage to destruction a_0. Its value is usually in the range d_0 = 0.2… 2.0 and determines the danger of the process. The smaller the value of d_0, the more intense the degradation process, the more dangerous it is. Therefore: U_i=c_i /d_0. With this in mind, we have:

$$N_\Sigma = \frac{d_0}{\Sigma\left(\frac{c_i}{N_i}\right)}. \tag{2}$$

In this form, a formula is obtained that coincides with the known formula for summing the damage taking into account the factor of nonstationarity. If there is no data on the influence of the shape of the block on the accumulation of damage, take d_0 =1. For several degradation processes acting on the element simultaneously, their relative duration is the same and is c_i =1.

3. The method of equivalence as a result of amalgamating the indicators of the stress-strain state

3.1. Equivalence by normal stresses

Along with the formation of the science of the strength of materials, the first theories of strength, designed for the CSS, emerged. In the 18th -19th centuries, six classical hypotheses were developed, which postulate that the destruction will occur when the complex indicator of the stress state reaches the critical values corresponding to uniaxial stretching (Fig. 2). If only the strength characteristics of the basic deformation process are known (B, Fig. 2), then the equivalence at normal stresses is a rather painstaking task. In the general case, to ensure the strength of the CSS, it is necessary to comply to 12 conditions (Collins (1981)).

The problem is simplified when the strength characteristics of the additional deformation process are known (A, Fig. 2), for example, in shear. Then the equivalent stress at the combined action of normal and tangential stresses is defined as:

$$\sigma_{eq} = k_\sigma \cdot \sigma. \tag{3}$$

Here k_σ is the load factor of the basic deformation process, which is the result of combining strength indicators in different deformation schemes.

Classical hypotheses were invented for static loading. But at least three of them are common in the case of multiaxial fatigue and are widely used today. Durability with mixed deformation, which corresponds to the CSS, is determined by the equation of the fatigue curve (S-N curve) for the base process:

$$N_{\Sigma eq} = k_N \cdot N_B = k_\sigma^{-m} \cdot N_B . \quad (4)$$

Here $m=m_\sigma$ is the slope indicator of the fatigue curve, expressed for normal stresses, and durability has a content of equivalent.

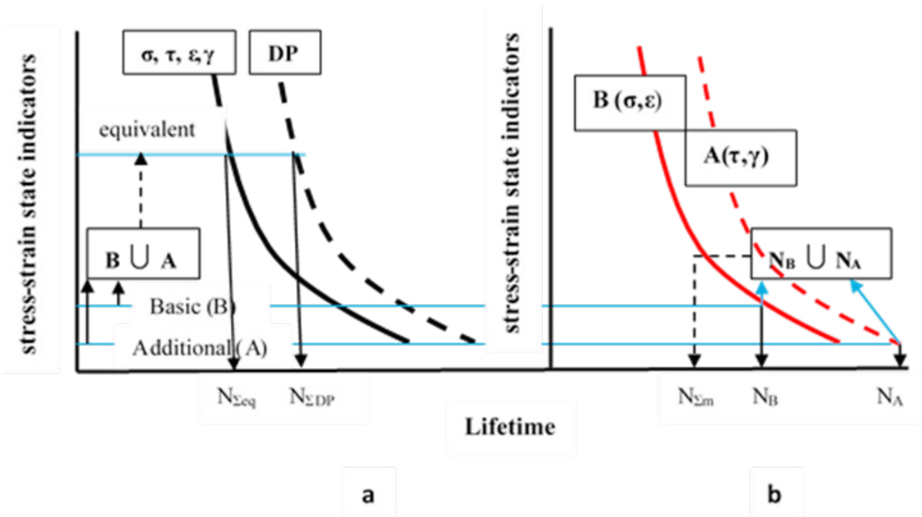

Fig. 2. (a) Schematic diagram for determining the durability of CSS N_Σ by equivalence; (b) Schematic diagram for determining the durability of CSS N_Σ by amalgamating RSI.

In general, the method of equivalence at normal stresses is based on the similarity of the behavior of indicators of static and cyclic strength. That is, it is expected that the ratio of yield strength and fatigue limit in tension and shear does not change. An ambiguous result arises when the cyclic basic process is influenced by an additional one, which parameters may be different in asymmetry. And this cannot be taken into account within the considered models.

3.2. Equivalence by tangential stresses and strains

With the development of instrumental methods for studying the materials microstructure, researchers have come to the conclusion that the tangential stresses are responsible for the appearance of fatigue cracks. Naturally, in such a situation there is a feasibility of equivalence by tangential stresses. In the end of 1950s, Findley proposed a damaging option (DP, Fig. 2, a), which is essentially an equivalent amplitude. This criterion is given in the following interpretation (Suman et al. (2015)):

$$DP_F = \tau_{aeq} = k_\tau \cdot \tau_a = \tau_a \cdot \left(1 + k_F \cdot \frac{\sigma_{\max}}{\tau_a} \right). \quad (5)$$

Here, the operating tangential stress is given in the amplitude τ_a, which emphasizes that the main load process is cyclic. The normal stress σ_{max} acts as a parameter of the additional load process. The maximum value indicates that the deterioration could have as static as cyclic nature. The properties of the material are taken into account through the coefficient (Kluger and Łagoda, (2016)):

$$k_F = \frac{3}{\chi} + \frac{3}{2} = 3 \cdot \left(\frac{\tau_R}{\sigma_R} - 0.5 \right). \tag{6}$$

The fatigue limit for bending conditions should be taken for the case of a combination of torsion and bending for the fatigue limit at normal stresses σ_R according to the recommendations (Kluger and Łagoda, 2016). By Eq. (9) ratio τ_R/σ_R=0.5. For bending conditions, the fatigue limit is at least 33% greater than the tensile fatigue limit (Heywood, 1962). According to the authors, this difference reaches 50-75%. That is, in fact, the ratio τ_R/σ_R=0.5, which makes it impossible to apply criterion Eq. (5) in a similar situation.

A more flexible Erickson criterion has been developed for high levels of normal and tangential stresses based on the Findley criterion (Erickson at el., 2008). It takes into account the asymmetry of the cycle and works well in common-mode and disproportionate load.

The concept of the critical plane, in which fatigue cracks arise, is one of the most authoritative in solving the problem of multiaxial fatigue. It was embodied in the criteria Fatemi-Socie (Fatemi & Socie,1988). Here, the damaging parameter corresponds to the shear deformation γ. Coefficient of deterioration κ_γ in this case refers to the shear deformation. The secondary multiplicative term of this equation is the ratio of the amplitude of the normal stress acting perpendicular to the critical plane to the yield strength σ_Y. Having a damaging parameter DP, it is possible to build DP-N curve instead S-N curve and use them for resource prediction. This procedure is not always effective, as the tightness for DP-N curve may be smaller than for S-N curve.

As a result of this brief analysis, the following remarks can be made, which are needed to understand further developments. 1. The equivalence methods of CSS do not give a clear answer for which load processes - static or cyclic, proportional or disproportionate - they are suitable. This problem is especially acute for users of strength models – designers and maintenance staff. 2. Models of multiaxial fatigue do not work when τ_R/σ_R=0.5. 3. Experimental testing of multi-axis fatigue models remains problematic, as it requires the creation of special test equipment (Marciniak et al., (2008), Ogawa et al., (2019)). Therefore, there are relevant methods and techniques that simplify the simulation of the CSS.

4. Multiaxial fatigue lifetime model

Regarding the combined action of two load processes that lead to normal and tangential stresses, Eq. (2) is transformed as (Fig. 2, b):

$$N_{\Sigma m} = N_{eq} = \frac{a \cdot N_B \cdot N_A}{c_B \cdot N_A + c_A \cdot N_B} = a \cdot N_m = k_N \cdot N_B. \tag{7}$$

In this case, the estimated durability $N_{\Sigma m}$ corresponds to the combined (mixed) load, and durability N_B and N_A correspond to the net load at the base (B) and additional (A) load process. Normal stress cycling can be taken as the basic process, and tangential stress cycling as additional. Then $N_B=N_\sigma$, $N_A=N_\tau$. From Eq. (7) and Fig. 2, b a fundamental difference is visible between traditional methods of equivalence (Fig. 2, a) and the proposed model. Here not CSS indicators amalgamate, but directly lifetime. That is, guided by the scheme of solving the problem of complex load (Fig. 1), the amalgamating occurs at level 1 in the equivalenting method (strength, Fig.1), and in the proposed model - at level 2-4 (lifetime - safety, Fig.1).

Relative duration of processes c_i is determined by their frequency f: $c_B/c_A = f_B/f_A$. For the basic process take c_B=1 and the resource is already calculated for its frequency. It should be noted that in contrast to non-stationary loading, at the combined loading the sum of value c_i not necessarily equal to one. That is, $\Sigma c_i \neq 1$. For in-phase proportional process $f_B=f_A$ and $c_B=c_A$=1. Taking into account the process frequencies, in the proposed model, the disproportionate load is considered as a load with a phase shift.

In Eq.(7) instead of the accumulated damage d_0, which takes into account the non-stationary load, the accumulated damage is applied a, which takes into account the complexity of the load or CSS. In the general case, the damage accumulated to the limit state will be $d=d_0{\cdot}a$. Value a depends on the type of processes that make up the combined load. If the experimental durability at the mixed loading is known $N_{\Sigma exp}$, then $a=N_{\Sigma exp}/N_m$.

The variability of the value of a is primarily related to the behavior of the function Eq. (2). The value of a depends on the ratio N_B/N_A. At $N_B/N_A\to 0$, that is $N_A>>N_B$ we have $N_\Sigma\to N_B$ and $a\to 1$. Otherwise, when $N_A/N_B\to 0$ ($N_B>>N_A$) , $N_\Sigma\to N_A$, also we have $a\to 1$. Between these extreme positions, when durability N_A and N_B one order, value a decreases to a minimum, then increases. We can present a function $a(N_B/N_A)$ as piecewise-linear. The ratio of durability N_B/N_A depends on the ratio of stresses of the basic and additional processes. In the experiments, this stress ratio was regulated by the coefficient of the shoulder γ_L. For samples of rectangular cross section, it is $\gamma_L=\sigma/3\tau$. Therefore the function $a(N_B/N_A)$ can be represented as a function $a(\gamma_L)$:

$$a = 1 + \alpha_B \cdot \gamma_L , \tag{8}$$

where α_B is the intensity of the changing in the marginal accumulated damage from the base process.

For non-phase load function $a(\gamma_L)$ must be adjusted with the parameter Itoh-Sakane P_{IS}, which is related to the nonproportionate coefficient (Kida et al. (1997)). It plays the role of the deterioration factor and similarly to Eq. (5) we have:

$$a = P_{IS}^{-m} \cdot (1 + \alpha_B \cdot \gamma_L). \tag{9}$$

If the additional loading process is static, then we have a situation $N_A>>N_B$, $N_\Sigma\to N_B$. But in this case, the value of a depends on the relative to the yield strength of the additional process $\bar{\sigma}$ or $\bar{\tau}$. According to the work (Wildemann et al. (2018)) α_B= - 0.5… - 0.8. However, in general, the effect of static additives is somewhat more complicated and depends on the type of deformation.

5. Prerequisites for studies of multiaxial fatigue under three-point bending

A number of features that are most clearly visible for prismatic samples were discovered by the authors when testing various steels for three-point bending. The influence was investigated for the distance between the supports of the sample (span length) on the fatigue resistance. This factor is characterized by the shoulder coefficient (multiplicity of span) γ_L as the ratio of the sample height h to half-span $L/2$. With its decrease and increase of the stress gradient, including along the length of the sample, the laws of crack growth change. They grow more intensively in the high-altitude direction with a decrease in the shoulder coefficient (Belodedenko et al. (2014)). Another feature of the behavior of materials in transverse bending is associated with an increase of the cyclic strength with a reduction of the span, if normal stresses are used as a criterion. The fatigue tests are evidenced about this for ductile steel 09G2 (ultimate strength σ_B=462 MPa, yield stress σ_Y=328 MPa, reduction of area ψ=0.56): with a decreasing of the shoulder coefficient from γ_L=2.5 to γ_L=1 fatigue limits σ_R, expressed in the maximum conditionally elastic stresses of the cycle increase by 20%. For thermo-resistant steels, for example, steel 40H (σ_B=1480 MPa, σ_Y=1180 MPa, ψ=0.43) also an increase of lifetime is observed, almost an order during with transition from γ_L=2 to γ_L=1.

If the fatigue curve is represented by the equation

$$\sigma \cdot N^{\frac{1}{m_\sigma}} = 10^{C_\sigma}, \quad \text{or} \qquad \tau \cdot N^{\frac{1}{m_\tau}} = 10^{C_\tau}, \tag{10}$$

then its parameters are as follows:

- steel 09G2 - m_σ=6, C_σ=3.75 (γ_L=2.5) ; m_σ=6, C_σ=3.83 (γ_L=1),
- steel 40H - m_σ=9.3, C_σ=3.55 (γ_L=2.0) ; m_σ=9.3, C_σ=3.60 (γ_L=1).

The abnormality of such behavior is difficult to explain from the positions of classical strength theories in which normal stresses are equivalented. The deterioration coefficient κ_σ is proportional to the ratio τ/σ. Its increasing leads to an increasing of equivalent stress σ_{eq}, that gives a reduction of lifetime $N_{\Sigma eq}$ (Fig. 2, a). For samples of rectangular forms ratio $(\tau/\sigma)=(1/3\gamma_L)$ is inverse to a value γ_L. So increasing γ_L in Eq. (3) leads to a decrease σ_{eq} and to increase $N_{\Sigma eq}$. In fact, the opposite picture is observed.

Instead of consider phenomenon easily explained from the standpoint of modern criterias in which tangential stress are equivalent. For example, according to the criterion Findley tangential equivalent stress τ_{eq} increases with value γ_L, that leads to a decrease of resulting lifetime $N_{\Sigma eq}$. That is actually observed. Therefore, the resulting curves of fatigue should be rebuilt for tangent stresses in the form Eq.(10).

Parameters of this equation are as follows:

- steel 09G2 - m_τ=6, C_τ=2.88 (γ_L=2.5) ; m_τ=6, C_τ=3.35 (γ_L=1),
- steel 40H - m_τ=9.3, C_τ=2.77 (γ_L=2.0) ; m_τ=9.3, C_τ=3.12 (γ_L=1).

From the given data it can be seen that when variety of values γ_L the incline of curves of fatigue remains: $m_\sigma = m_\tau = m$. In addition, behavior of material becomes predictable in support of multiaxial fatigue: the function of the fatigue limit from the shoulder coefficient $\tau_R(\gamma_L)$ monotonically decrease (Fig. 3). Therefore, the task of fatigue tests in a three-point bend appears to obtain the equation of this function. In this case, it is possible to predict the resource at CSS.

All these considerations can become unable, if we remember that theoretically material in the destructive intersection of the sample is in a state of pure bending and does not meet the conditions CSS. But it should also be noted that on both sides of the median intersection operate the maximum shear stress. They affect to the birth and development of cracks. In addition, as clarified by the last research, the crack occurs at a distance from the median plane or root notch tip, where destruction is expected (Bressan et al. (2019)).

6. Experimental verification of the model under three-point bending

In addition to the previously presented studies, fatigue tests of the same samples were performed for γ_L=0.5 and γ_L=5. The tests were performed on a hydropulsator with asymmetry R_σ=0.1 and frequency 15 Hz. The parameters of the fatigue curves were obtained C_σ and C_τ. From the obtained functions of these parameters from the value γ_L the trends are viewed that listed in the previous section. For the function $C_\sigma(\gamma_L)=C_{\sigma\gamma}$ an analytical form could not be finded. However, function $C_\tau(\gamma_L)=C_{\tau\gamma}$ with a high correlation can be represented by a second degree polynomial. A similar function can be represented by the dependence of the limit of fatigue $\tau_{R\gamma}$ from the value γ_L (Fig. 3).

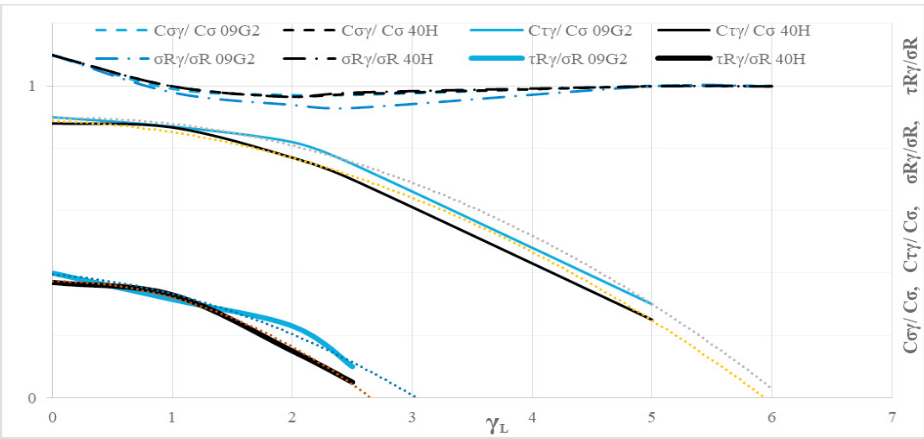

Fig. 3. Parameter functions $C_{\sigma\gamma}$, $C_{\tau\gamma}$, fatigue limits $\sigma_{R\gamma}$, $\tau_{R\gamma}$, attributed to the parameters of fatigue resistance at pure bending (γ_L=5) C_σ and σ_R.

To summarize the test results, the parameters of fatigue resistance at CSS attributed to the parameters obtained at pure bending, for which the results are taken at γ_L =5: σ_R and C_σ (Fig. 3). These results show that the function $\tau_{R\gamma}$ for two selected steels can be represented by a single equation. The first free term of this equation is the ratio of fatigue limits for pure types of deformation $\tau_R/\sigma_R=0.385$. Therefore, the presented equation can be generalized as:

$$\left(\frac{\tau_R}{\sigma_R}\right)_\gamma = \frac{\tau_R}{\sigma_R}\cdot k_\tau = \frac{\tau_R}{\sigma_R}\cdot\left(1-\delta_1\cdot\gamma_L-\delta_{11}\cdot\gamma_L^2\right) \tag{11}$$

In Eq. (11) deterioration factor κ_τ expressed as a polynomial of the second degree. Equation parameters for both tested materials δ_I=0.028, δ_{II}=0.116. It is true when γ_L<2.8. Index γ indicates the mixed nature of the deformation,

Having data on $C_{\tau\gamma}$, we can get the actual lifetime under mixed load $N_{\Sigma exp}$, then we can find a change in the accumulated damage a. Parameters of total piecewise linear function $a(\gamma_L)$ Eq. (8): a_B= -0.75 (at γ_L=1-2), a_B=0.3 (at γ_L=2.5-5).

7. Conclusions

The ability of the rule to amalgamate resource indices of safety to predict lifetime in multi-axial fatigue has been confirmed. In this case, the combined load is considered as a composition of individual simple processes of cyclic deformation with its parameters. This gives us an opportunity to use fatigue resistance characteristics for simple (pure) types of deformation without resorting to unique and complex test techniques. The using of the security index method gives an opportunity to assess the resource for any level of reliability. The proposed model allows to take into account the shape of the cycle and the type of process.

An explanation is found for the behavior of materials in transverse bending under conditions of changing of the coefficient of the shoulder. In this case, the resistance to multi-axial fatigue is controlled by criteria which are based on tangential stresses. The possibility of obtaining the parameters of the model of multi-axial fatigue when tested for three-point bending under conditions of variation of the multiplicity of the span is confirmed.

References

Belodedenko, S.V., Bilichenko, G.M., Hrechanyi, O.M., Ibragimov, M.S., 2019. Application of risk-analysis methods in the maintenance of industrial equipment. Procedia Structural Integrity 22, 51–58.

Belodedenko, S., Grechany, A., Yatsuba, A., 2018. Prediction of Operability of the Plate Rolling Rolls Based on the Mixed Fracture Mechanism. Eastern-European Journal of Enterprise Technologies 1(7), 4–11.

Belodedenko, S.V., Goryany, V.M., Buch, J., Yatsuba, A. V., 2014. Prediction of the Serviceability of Sheet Rolls. Strength Mater 46, 654–659.

Bressan, S., Ogawa, F., Takamoto, I., Berto, F., 2019. Influence of notch sensitivity and crack initiation site on low cycle fatigue life of notched components under multiaxial non-proportional loading. Frattura ed Integrità Strutturale 47, 126-140.

Collins, J.A., 1981. Failure of Materials in Mechanical Design: Analysis, Prediction, Prevention. Wiley & Sons, pp. 630.

Erickson, M., Kallmeyer, A.R., Vanstone, R.H., Kurath, P., 2008. Development of a multiaxial fatigue damage model for high strength alloys using critical plane methodology. J. Eng. Mater. Technol. 130, 1–9.

Fatemi, A., Socie, D., 1988. A critical plane approach to multi-axial fatigue damage including out-of-phase loading. J. Fat. Fract. Eng. Mater. Struct. 11, 149-165.

Heywood, R.B., 1962. Designing against fatigue. Chapman and Hall, London, pp. 436.

Kida, S., Itoh, T., Sakane, M., Ohnami, M., Socie, D. F., 1997. Dislocation Structure and Non-proportional Hardening of Type 304 Stainless Steel. Fatigue Fract. Engng Muter. Struct 20(10), 1375–1386.

Kluger, K., Łagoda, T., 2016. Fatigue Life Estimation for Selected Materials in Multiaxial Stress States with Mean Stress. Journal of theoretical and applied mechanics 54(2), 385-396.

Marciniak, Z., Rozumek, D., Macha, E., 2008. Fatigue lives of 18G2A and 10HNAP steels under variable amplitude and random non-proportional bending with torsion loading. International Journal of Fatigue 30, 800–813.

Ogawa, F., Shimizu, Y., Bressan, S., Morishita, T., Itoh, T., 2019. Bending and Torsion Fatigue-Testing Machine Developed for Multiaxial Non-Proportional Loading. Metals 9, 1115.

Rakhmanov, S.R., Belodedenko, S.V., Kasyanov, N.V., 2020. Assessment of the Reliability Functioning of the Beds Working Stands of TPA 350 Piercing Mills According to the Stress-strain State. Steel 10, 48–53. (in Russian).

Sosnovsky, L.A., 1987. Statistical mechanics of fatigue fracture. Science and technology, Minsk, pp. 288. (in Russian).

Suman, S., Kallmeyer, A., Smith, J., 2016. Development of a multiaxial fatigue damage parameter and life prediction methodology for non-proportional loading. Frattura ed Integrità Strutturale 38, 224–230.

Wildemann, V.E., Tretyakov, M.P., Staroverov, O.A., Yankin, A.S., 2018. Influence of the Biaxial Loading Regimes on Fatigue Life of 2024 Aluminum Alloy and 40CRMNMO Steel. PNRPU Mechanics Bulletin 4, 169–177.